CPSC 421/501 Nov. 6

Last time: $NP = \bigcup_c NTIME(n^c)$

Def: L, (language over Σ), is **NP-complete** if

① $L \in NP$

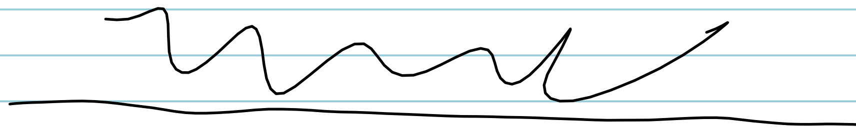
② $L' \in NP$, $L' \leq L$

deterministic poly time algorithm

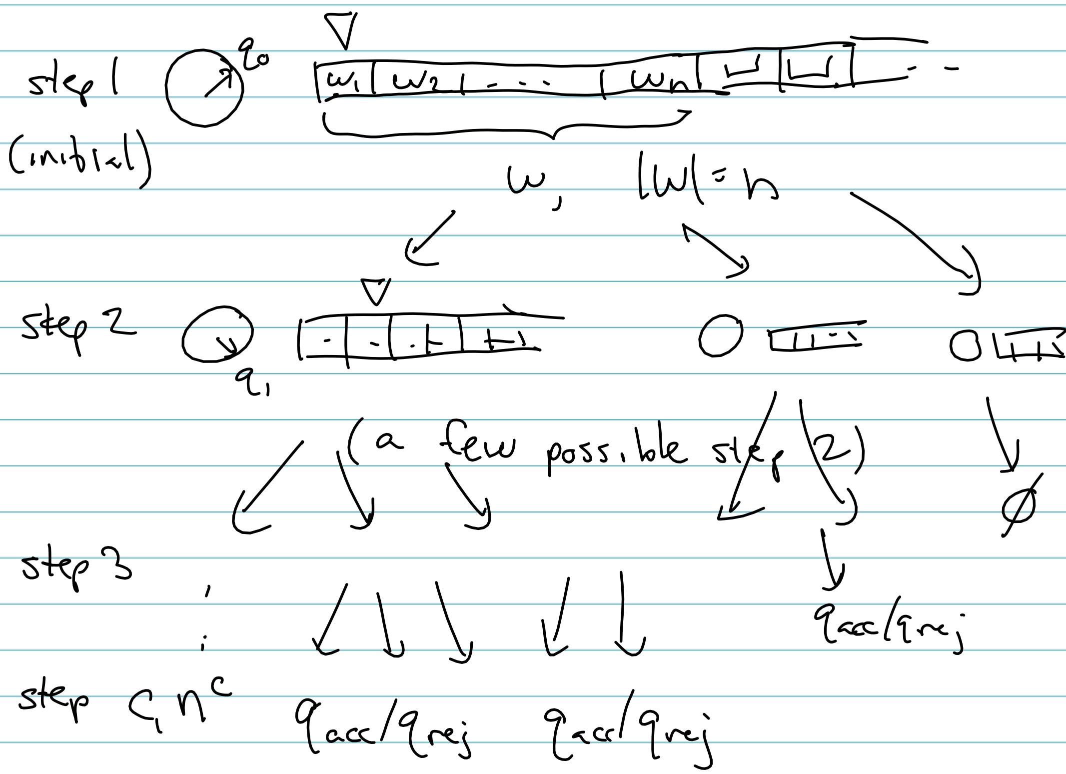
Intuition: L is NP-complete, then L is "a hardest problem" in NP up to polynomial time reductions.

(1) Cook-Levin: SAT is NP-complete.

$\Rightarrow$ SUBSET-SUM is NP-complete, . . .



M' :



w is accepted by M' $\iff$ on at least one computation path we reach q_{acc}

Cook-Levin: If $L' \in NP$, i.e. there is a Non-det T.M (NTM) that decides L' in time $Order(n^c)$ then $\forall w \in (\Sigma')^*$ (L' is a language over Σ') there is a "reduction"

takes $w \in (\Sigma')^*$ $\longrightarrow$ $f(w) \in (\Sigma_{SAT})^*$

f computed by a poly time (deterministic) algorithm

s.t.

$$w \in L' \iff f(w) \in SAT$$

Pf: Let M' be NTM that decides L' in time order n^c, c constant.

(Recall: for M', NTM (non-det Turing m.)

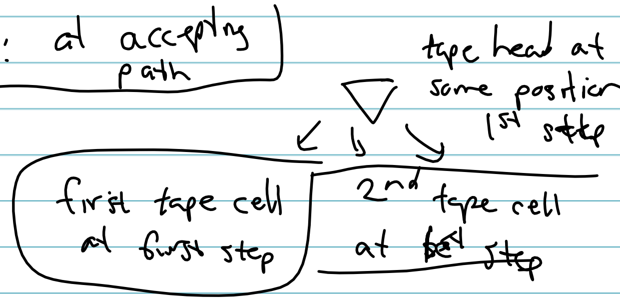
$\delta : Q \times \Gamma \to$ Set of Subsets $(Q \times \Gamma \times \{L, R\})$)

It's OK for $\delta(q_1, \gamma_{20}) = \phi$

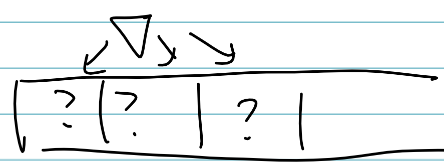
w is accepted: an accepting path

We have

1st step — first state

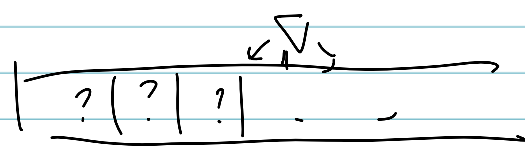


2nd step — state



⋮

(order n^c) ($c \cdot n^c$) state



Such that

① at step 1: state q_0
see w_1 on cell 1
" w_2 " " 2
⋮

AND tape head is on cell #1

(2) at step $c'n^c$: state q_{acc}

AND

(3) at step s, $1 \le s \le c'n^c - 1$ transition from step s to step $s+1$ is permitted by M'

=

Make this a formula...

$Q = \{1, \ldots, a\}$
$\Sigma = \{1, \ldots, b\}$
$\Gamma = \{1, \ldots, \gamma\}$
} for M'

⋮

$X_{ij} = \begin{cases} \text{True if at time } i \text{ we are in state } j \\ \text{False if we are not} \end{cases}$

$X_{11}, \ldots, X_{1a}$: one will be True, all others False

tells us: at time $i=1$, what state are we in

$$Y_{ijk} = \begin{cases} T & \text{if at time } i\text{, cell } \#j \text{ is equal to } k \\ \text{False} & \text{otherwise} \end{cases}$$

$$Z_{ij} = \begin{cases} T & \text{if tape head at time } i \text{ is at cell } \#j \\ F & \text{otherwise} \end{cases}$$

X_{ij}: $i = 1, 2, \ldots, c'n^c$ } time
$j = 1, \ldots, a$

Y_{ijk}: $i = 1, 2, \ldots, c'n^c$ ← time
$j = 1, 2, \ldots,$ max tape cell we reach at time i $\leq i+1 \leq c'n^c+1$
$k = 1, \ldots, r$

Z_{ij}: $i = 1, \ldots, c'n^c$
$j = 1, \ldots, c'n^c+1$

Rem: Just # variables is $\text{Order}(n^{2c})$