Oct 28, 2015
CPSC 421/501

- Show acceptance is undecidable (abstract)
- Midterm topics, sample problems
- Homework coming back today

Midterm on Oct 30
Covers up to Oct 16 -
showed A_{TM} is undecidable (but recognizable)

Today following §6 of Handout
(Midterm covers §1-4 of handout)

"Abstract away" what we don't need

Axioms:

- sets $\mathcal{P}, \mathcal{I}$, Result: $\mathcal{P} \times \mathcal{I} \to \{\text{yes}, \text{no}, \text{nohalt}\}$

(denote $P[i]$)

- have EncodeBoth: $\mathcal{P} \times \mathcal{I} \to \mathcal{I}$ denoted $\langle P, i \rangle$, there is a $U \in \mathcal{P}$ s.t.

$\text{Result}(U, \langle P, i \rangle) = P[i]$

or $U[\langle P, i \rangle] = P[i]$

- have EncodeProg: $\mathcal{P} \to \mathcal{I}$ denote $\langle P \rangle$ s.t. for any $P \in \mathcal{P}$ $\exists$ $P' \in \mathcal{P}$ s.t.

$$** \quad P'[\langle Q \rangle] = P[\langle Q, \langle Q \rangle \rangle] \text{ for all } Q$$

- for all $P \in \mathcal{P}$, $\exists$ $P' \in \mathcal{P}$ s.t.

$$* \quad \boxed{P'[i] = \neg P[i]}, \text{ where}$$

$\neg$ yes = no, $\neg$ no = yes, $\neg$ nohalt = nohalt

$*$: add a bit of code at the end

$**$: " " " " " beginning:

input is $\langle Q \rangle \to$ find the "string" for $\langle Q, \langle Q \rangle \rangle$

run P on $\langle Q, \langle Q \rangle \rangle$

$*$ $\Rightarrow$ negation

$**$ $\Rightarrow$ self-referencing..

Now claim:

$$A_{\mathcal{P}} = L_{yes} = \{ \langle P, i \rangle \mid P[i] = \text{yes} \}$$

is recognizable (by U)
but undecidable:

<u>Proof</u>: Say H <u>decides</u> $A_{\mathcal{P}} = L_{yes}$.

i.e. $H[i] = \begin{cases} \text{yes} & \text{if } i \in A_{\mathcal{P}} \\ \text{no} & \text{otherwise} \end{cases}$

Let:

$$(***) \quad D[\langle Q \rangle] = \neg H[\langle Q, \langle Q \rangle \rangle].$$

Claim: $D[\langle D \rangle]$ = yes or no,

but we get a contradiction either way!

Midterm topics:

Handout §1–4

- Paradoxes
  - Self-referencing with negation
    - Set of all sets that don't contain themselves (Russell's paradox)
  - Thm: $|S| < |\text{Power}(S)|$
    If $f: S \to \text{Power}(S)$ that is surjective: let
    $T = \{ s \in S \mid s \notin f(s) \}$
  - Berry Paradox (Exercises: moo two)
  - "This is a lie"
- Countable versus uncountable
- {Languages} usually uncountable

By

$D[\langle Q \rangle] = \text{yes}$ if

$\neg H[\langle Q, \langle Q \rangle \rangle] = \text{yes}$

$H[\langle Q, \langle Q \rangle \rangle] = \text{no}$

Q on input $\langle Q \rangle = Q[\langle Q \rangle]$
$= \text{no}$

D on input $\langle Q \rangle = \text{yes} \Rightarrow$
Q on input $\langle Q \rangle = \text{no}$

So D on input $\langle D \rangle = \text{yes}$
$\Rightarrow D$ on input $\langle D \rangle = \text{no}$

D on input $\langle D \rangle = \text{no} \Rightarrow$ similar contradiction

- {Programs} or {Algorithms} usually countable

$\boxed{\text{- Ch. 3 Turing machines, multi-tape Turing machines}}$

- ~~Ch 4~~: PALINDROME: takes order n^2 one tape, order n 2-tape

Ch 4: Counting, A_{TM} is recognizable, but undecidable