decidable = always halts (in finite # steps)

recognizable: L is recognizable if there is a T.m, M, s.t. L, M are over alphabet Σ, and for $w \in \Sigma^*$

$w \in L \iff M$ accepts w (reaches q_{acc})

alternatively

$w \notin L \iff M$ reject w or doesn't halt on w

Ch 4, Ch 5 reductions

A_{TM} = Turing machine acceptance problem

$HALT_{TM} = \{\langle M, w\rangle \mid M \text{ halts on input } w\}$

=

Last time:

$A_{TM} = \{\langle M, w\rangle \mid M\text{, on input } w\text{, reaches } q_{acc}\}$

not decidable [self-referential proof by contradiction]

however recognizable

A_{TM} is undecidable (recognizable)

$\overline{A_{TM}}$ is unrecognizable

=

Ch 5 - $HALT_{TM}$, $L_{\text{state\_isn't\_reached}}$, ...

Ch. 7 - deciding ~> deciding in poly time

=

Question: What is $\overline{A_{TM}}$?

$\overline{A_{TM}} = \{w \in \Sigma^* \mid w \text{ is not of the form } \langle M, w\rangle \text{ s.t. } M \text{ accepts } w\}$

Also: if L is not decidable but is recognizable, then $\overline{L}$, i.e. L^{comp}, i.e. $\Sigma^* \setminus L$ is not recognizable (!!!)

=

{Turing machines} is countable

{languages} is uncountable

$\Rightarrow$ (wide sense) there are {undecidable, unrecognizable, ...} languages

$HALT_{TM} = \{\langle M, w\rangle$ s.t. M halts on $w\}$

i.e. halt = eventually reach q_{acc} or q_{rej}

I

Recognize $HALT_{TM}$:

given word

- check if it's of the form $\langle M, w\rangle$
- simulate M on w with Univ T.M.
- if simulations halts, we accept $\langle M, w\rangle$

$= \{u$ s.t. u isn't of the form $\langle M, w\rangle\} \cup \{u$ is of the form $\langle M, w\rangle$ but M does not accept $w\}$

Not_{TM} Not $Accept_{TM}$

$u \in \Sigma^*$, u is one of

$A_{TM} = \{\langle M, w\rangle \mid M \text{ accepts } w\}$

$NotA_{TM} = \{\langle M, w\rangle \mid M \text{ does not accept } w\}$

$NotAtAll_{TM} = \{u$ is not of the form $\langle M, w\rangle\}$

Imagine there is a T.m. I that decides $HALT_{TM}$.

Given $\langle M, w\rangle$: make $\hat{M}$ and $\hat{w}$ out of M and w:

1. $\hat{w} = w$
2. $\hat{M}$ ~~does~~ is built like M except that if M rejects w then $\hat{M}$ loops infinitely

Believe $HALT_{TM}$ is undecidable...

We know A_{TM} is undecidable.

To prove $HALT_{TM}$ is undecidable:

$$A_{TM} \underset{\text{reduction}}{\le} HALT_{TM}$$

Reduction: ~~If~~ Given L language we want to see if $L \in A_{TM}$

Claim: If we could solve $HALT_{TM}$, we could solve A_{TM}...

$\langle M, w\rangle \in A_{TM}$

$\iff \langle \tilde{M}, w\rangle \in HALT_{TM}$

formal notion of reduction: I.e. there is a finite time computable $f: \Sigma^* \to \Sigma^*$ s.t.

$w \in A_{TM} \Rightarrow f(w) \in HALT_{TM}$

$w \notin A_{TM} \Rightarrow f(w) \notin HALT_{TM}$

=

$A_{TM} \leq HALT_{TM}$

we can <u>reduce</u> A_{TM} to $HALT_{TM}$

$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

$\tilde{M} = (\tilde{Q}, \Sigma, \Gamma, \tilde{\delta}, q_0, q_{acc}, q_{rej})$

s.t. δ and $\tilde{\delta}$ are same except

$\delta: (\;,\;) \longmapsto (q_{rej}, \;,\;)$

$\tilde{\delta}: (\;,\;) \longmapsto (q^{fake}_{rej}, \;,\;)$

$\tilde{\delta}: (q^{fake}_{rej}, \;) \longmapsto (q^{fake}_{rej}, \;,\;)$

$\tilde{Q} = Q \cup \{q^{fake}_{rej}\}$

M accepts w: $\tilde{M}$ accept w

M rejects w: $\tilde{M}$ doesn't halt

M doesn't halt on w: $\tilde{M}$ doesn't halt

<u>If</u>

$$A_{TM} \leq HALT_{TM}$$

but A_{TM} is not decidable then $HALT_{TM}$ cannot be decidable.

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More generally: $L_1 \leq L_2$ and L_2 decidable $\Rightarrow L_1$ decidable

equiv L_1 not decidable $\Rightarrow L_2$ not decidable

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