

A_{TM} can be recognized, but not decided.

Why?

Recognized: given $\langle M, w \rangle$, we can simulate M on input w by some fixed machine, a "Universal Turing Machine"

Can't be decided: If H decided

A_{TM} , then get contradiction:

Consider D takes input w , if $w = \langle M \rangle$ then D

If D rejects $\langle D \rangle$ then

$\langle D, \langle D \rangle \rangle \in A_{TM}$

$\Rightarrow D$ accepts $\langle D \rangle$.

(Contradiction.)

"Turing machines can speak about themselves"

$w = \langle D \rangle \rightsquigarrow \langle D, \langle D \rangle \rangle$

↑
description of D

↑
string expresses D

$$A_{TM} = \{ \langle M, w \rangle \mid M \text{ accepts } w \}$$

=

$$L_{\text{visit_state}} = \{ \langle M, w, q \rangle \mid \begin{array}{l} \text{on input } w \\ M \text{ reaches } q \end{array} \}$$

(equivalent of dead code)

=

$$L_{\text{dead_code_in_C}}$$

$$= \{ \langle p, i \rangle \mid \begin{array}{l} p \text{ is a C-program} \\ i \text{ is an input to } C_p \\ p \text{ has at least one} \\ \text{line that is never visited} \end{array} \}$$

=

...

~~accepts~~ accepts $w = \langle M \rangle$ if

$\langle M, \langle M \rangle \rangle \notin A_{TM}$ and

reject $w = \langle M \rangle$ if

$\langle M, \langle M \rangle \rangle \in A_{TM}$.

=

Consider D on input $\langle D \rangle$:

D decides: accepts or rejects

(in some finite time)

If D accepts $\langle D \rangle$, then

$\langle D, \langle D \rangle \rangle \notin A_{TM} \Rightarrow D$ does not accept $\langle D \rangle$

$$\bar{L} = \Sigma^* \setminus L$$

$$= \{w \in \Sigma^* \mid w \notin L\}$$

then if L and \bar{L} can be recognized, then they both can be decided.

Why? $w \in \Sigma^*$

Is w in L or in \bar{L}

M_1 { $\begin{matrix} \text{O} \dashrightarrow & M_1 \text{ takes } w \text{ as} \\ \text{O} \dashrightarrow & \text{input, and accepts} \\ \text{O} \dashrightarrow & w \text{ in finite time} \\ & \text{if } w \in L \end{matrix}$

Since A_{TM} can be recognized but not decided,

$$\bar{A}_{TM} = \text{complement of } A_{TM}$$

$$= \{w \in \Sigma^* \mid w \text{ is not in } A_{TM}\}$$

cannot be recognized.

Proof! If A_{TM} and \bar{A}_{TM} can be recognized, then they can both be decided...

Generally: If $L \subset \Sigma^*$ and

Otherwise: If we have

$L_1, L_2 \subset \Sigma^*$, more generally

$L_1, L_2, L_3, L_4, L_5 \subset \Sigma^*$

run step 1 of M_1 ← $L_i \leftrightarrow$ machine M_i

run step 2 of M_2

⋮

run step 1 of M_5

run step 2 of M_1

... step 2 of M_2

step 2 M_5

run step 3 of M_1

⋮

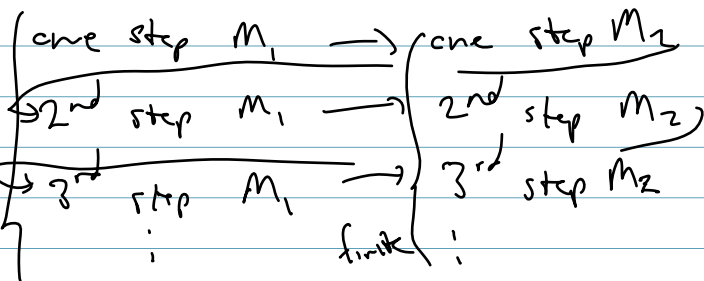
M_2 that if $w \in L$, i.e. $w \in \bar{L}$

step 1 $\text{O} \dashrightarrow$ then M_2

step 2 $\text{O} \dashrightarrow$ in a finite amount of time

accepts w .

Given w : run



If $w \in L$

If $w \in \bar{L}$

