We standardized graphs...
Now we have to standardize
Turing machines...

Say: $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej}, \sqcup)$

Insist:

$Q = \{1, \dots, a\}$

$1 = q_0$, $2 = q_{acc}$, $3 = q_{rej}$

$(a \geq 3)$.

$\Sigma = \{1, \dots, b\}$

$\Gamma = \{1, \dots, b, b+1, \dots c\}$

$\sqcup \Leftrightarrow b+1$.

Ch 4:

Sipser's textbook:

$A_{TM} = \{\langle M, w\rangle \mid M$ is a Turing machine, w is an input to M, w is accepted by $M\}$

A_{TM} "acceptance problem"

Sipser-halting-problem

$\langle M, w\rangle$ a description of M, w ...

Recall: $3COLOR = \{\langle G\rangle \mid G$ is 3-colorable$\}$

G, graph, usually: (V, E)

V, E sets ... too many

A Turing machine:

set Q

set $\Sigma = \{1, \dots, b\}$

set Γ

values $\delta(1^{st}$ state, 1^{st} letter$)$

$\delta(1^{st}$ state, 2^{nd} letter$)$

$\vdots$

$|Q| \times |\Gamma|$ values of δ

Any T.M. becomes an element of $\{0, 1, \dots, 9, \#, L, R\}^*$

Input: element of Σ^*

Turing machine:

Give a, b, c $(c \geq b+1)$

and δ:

$\delta: (1,1) \mapsto (q, \gamma, L$ or $R)$

As a word over

$\{0, 1, \dots, 9, \#, L, R\}$

$\underbrace{1\ 2\ 7}_{a\ b\ c}\ \#\ 5\ \#\ 8\ \#$ $\underbrace{\text{specify } \delta}$

$\delta(1,1)\ \#\#$

$\delta(1,2)\ \#\#$

$\vdots$

$\delta(1,1) = 13\#7\#L$

"M <u>decides</u> L vs.

M <u>recognizes</u> L"

=

$L = \text{PRIMES} \subset \{0,1,\ldots,9\}^*$

we have Algorithm(s) to say given $w \in \{0,1,\ldots,9\}^*$ if w represents a prime, our algorithm say "yes" and halt; if w doesn't our algorithm says "no" and halts. <u>Deciding</u>

Turing machine ↓ input

$\langle M, w \rangle$

a in base 10 # b in base 10 #

c in base 10 #

$\delta(1,1)$ ## $\delta(1,2)$ ## ... ##

$\delta(a,c)$ ## 1st letter of w #

2nd letter of w # ... #

last letter of w ##

↑ this is the end...

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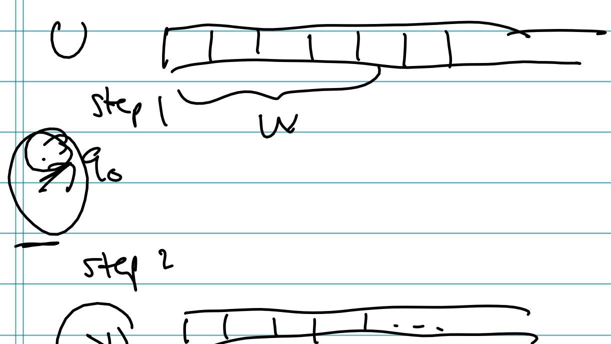
$A_{TM} = \{\langle M, w \rangle \mid M \text{ accepts } w\}$ is "Turing-recognizable"

$\langle M, w \rangle$ = long string over $\{0,1,\ldots,9,\#,L,R\}$

Universal Turing machine U:

U is given $\langle M, w \rangle$



U

step 1 $\underbrace{\qquad}_{w}$

q_0

step 2

step 3 → step 4 → ...

If an algorithm always halts, then it is a <u>decider</u>.

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Ch 4:

$A_{TM} = \{\langle M, w \rangle \mid M \text{ halts on } w\}$

is recognizable, but not decidable.

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Proof that A_{TM} is recognizable:

$\boxed{\text{One line: Simulate } M \text{ on } w \ldots}$

"Universal Turing Machine"

takes $\langle M, w \rangle$ as input, "simulates"