Last time:

Any language decidable by a k-tape machine in time $O(n^r)$ can be decided by a 1-tape TM in time $O(n^{2r})$. Can we do better than $2r$?

We knew PALINDROME can be done time $O(n)$ 2-tape TM.

We stated a theorem: for any TM, M, deciding PALINDROME, there is a constant $C = C_M$ s.t. M takes time $\geq n^2/C_M$

Hence, for general r, i.e. $r=1$, can't do anything less than n^{2r}/C, $C = C_M$

Now should see: $\exists$ a universal Turing machine (Ch. 4, Sipser)

2 steps:

1. k-tape TM's are as powerful as C, Java, Lisp, Python, ...
2. We can simulate $\langle M, w \rangle$ the TM, M, on input w

Meaner than MAJORITY_OF_ONES

Nicer than SOLVE_QUADRATIC

$x^2 - 3x + 5 = 0$ also need $\sqrt{\cancel{w}}$ $\sqrt{n}$

LANGUAGE:

$\Sigma = \{0, \ldots, 9, \#\}$

$SQRT = \{ w \in \Sigma^* \mid w = w_1 \# w_2 \# w_3$

s.t. w_1 represents integer

w_2 " "

$w_3 \in \{0, \ldots, 9\}$

s.t. the w_2-th digit of $\sqrt{w_1}$ is w_3 $\}$

High-level (algorithm) T.m. description:

find $\sqrt{w_1}$'s 1st digit

— — — 2nd digit

— —

1. find $n \in \mathbb{Z} = \{\text{integers}\}$ s.t. $n^2 \leq w_1 < (n+1)^2$
2. find $d_1 \in \{0, \ldots, 9\}$ s.t. $\sqrt{w_1} = n.d_1 \ldots$

— — . d_2 — — —

| 7 | 7 5 | 2 1 #

tape 2 : | 1 |

0, …, 9

compute 1^2

then

| 7 | 75 | 21

| 2 | compute 2^2

multiply

3rd tape →

input tape 1

tape 2 : running $\sqrt{w_1}$ approx

tape 3 : guess next digit, another few tapes for squaring

Step 1: Medium-Level (Implementation)

w_1 # w_2 # w_3 ⊔

$\sqrt{7,75\,21}$ = blah. blah…

$w_1 = t_\ell \ldots t_3\, t_2\, t_1$

$t_1 \to$ two-digits

$t_2 \to$ " "

⋮

t_ℓ has one or two digits

∇

=

Graph G

⟨G⟩ description of G

$G = (V, E)$ V set, E set

V has many possibilities…

⟨G⟩ ? insisted $V = \{1, \ldots, n\}$

to describe V : 1, 2, 971, 31528, …

How do we square

tape 3 | 2 | 1 | 7 . 6 5 3

$$
\begin{array}{r}
217.653 \\
\times\ 217.653 \\
\hline
.2959
\end{array}
$$

tape 4

2nd row

tape 5 multiply for each row

tape 6 running total.

Insist:

$Q = \{1, \ldots, a\}$

q_0, q_{acc}, q_{rej}

insist $q_0 =$ state 1

$q_{acc} =$ " 2

$q_{rej} =$ " 3

$\Sigma = \{1, \ldots, b\}$

$\Gamma = \{1, \ldots, b, b+1, \ldots c\}$

$\sqcup$

So

V is described by $\{0, 1, \ldots, 9\}^*$

In this way

$\langle G \rangle$, G is essentially general,

but we require V special

$\langle G \rangle \subset \{0, 1, \ldots, 9, \#\}^*$

M, Turing machine

$\langle M \rangle = \underset{\text{of } M}{\text{description}} = \langle Q, \Sigma, \Gamma, \delta, \ldots \rangle$

31C

$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

$\{1, \ldots, a\}$ $\{1, \ldots, c\}$

$\delta : (1, 1) \longmapsto (\quad , \quad , L/R)$

$\delta : (1, 2) \longmapsto (\quad , \quad , L/R)$

$\vdots$

$L = 1$

$R = 2$