

Hence, for general  $r$ , i.e.  
 $r=1$ , can't do anything less  
 than  $n^{2r}/C$ ,  $C=C_M$

=  
 How should see:  $\exists$  a universal  
 Turing machine (Ch. 4, Sipser)

=  
 2 steps:

①  $k$ -tape TM's are as  
 powerful as C, Java, Lisp, Python,  
 ...

② We can simulate  
 $\langle M, w \rangle$  the TM,  $M$ , on  
 input  $w$

s.t. the  $w_2$ -th digit  
 of  $\sqrt{w_1}$  is  $w_3$

=  
 High-level (algorithm) T.M.  
 description:

find  $\sqrt{w_1}$ 's 1st digit  
 - - - 2nd digit

① find  $n \in \mathbb{Z} = \{\text{integers}\}$   
 s.t.  $n^2 \leq w_1 < (n+1)^2$

② find  $d_1 \in \{0, \dots, 9\}$  s.t.  $\sqrt{w_1} = n.d_1 \dots$   
 - - -  $d_2$

Last time:

Any language decidable by a  
 $k$ -tape machine in time  $O(n^k)$   
 can be decided by a 1-tape TM  
 in time  $O(n^{2r})$ . Can we do  
 better than  $2r$ ?

=  
 We knew PALINDROME can be  
 done time  $O(n)$  2-tape TM.  
 We stated a theorem: for any  
 TM,  $M$ , deciding PALINDROME, there  
 is a constant  $C=C_M$  s.t.  $M$  takes  
 time  $\geq n^2/C_M$

Meaner than MAJORITY\_OF\_ONES  
 Nicer than SOLVE\_QUADRATIC

$$x^2 - 3x + 5 = 0 \quad \text{also need } \sqrt{\frac{9}{4} - 5}$$

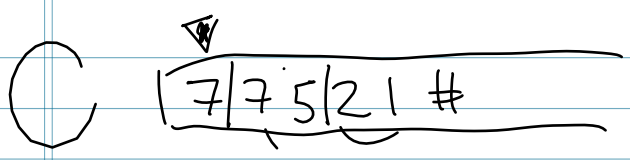
$$\sqrt{n}$$

LANGUAGE:

$$\Sigma = \{0, \dots, 9, \#\}$$

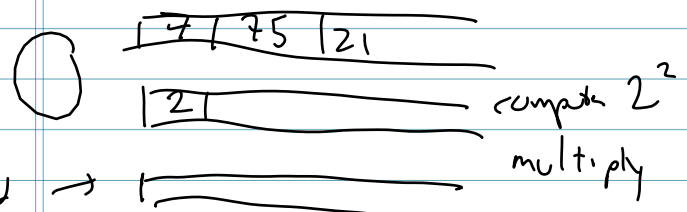
$$\text{SQRT} = \{w \in \Sigma^* \mid w = w_1 \# w_2 \# w_3$$

s.t.  $w_1$  represents integer  
 $w_2$  " " "  
 $w_3 \in \{0, \dots, 9\}$



type 2: [ ]  
 compute  $1^2$

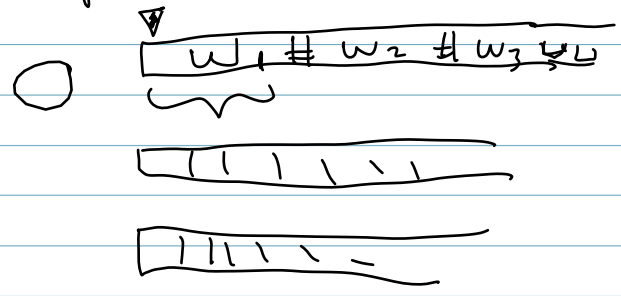
then



3rd type

input tape  
 type 2: running  $\sqrt{w_i}$  approx  
 type 3: guess next digit, another few tapes for squaring

Step 1: Medium-Level (Implementation)



$\sqrt{7, 75, 21} = \text{blah. blah...}$

$w_1 = t_2 \dots t_3 t_2 t_1$

$t_1 \rightarrow$  two-digits

$t_2 \rightarrow$  " "

:

$t_2$  has one or two digits



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Graph  $G$

$\langle G \rangle$  description of  $G$

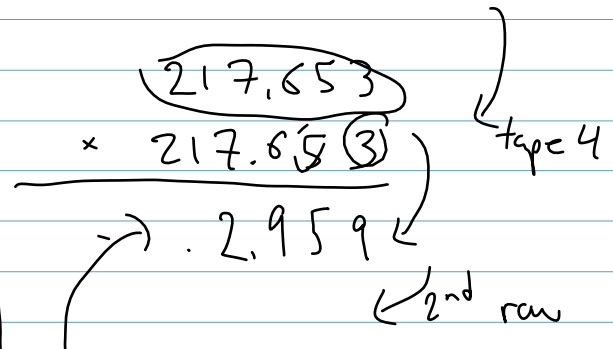
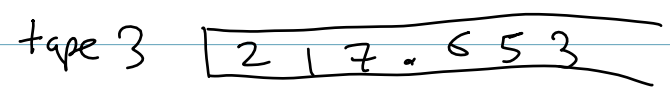
$G = (V, E)$   $V$  set  
 $E$  set

$V$  has many possibilities...

$\langle G \rangle$ ? instead  $V = \{1, \dots, n\}$

to describe  $V$ : 1, 2, 971, 31528, ...

How do we square



tape 5 multiply for each row

tape 6 running total.

Insist:

$$Q = \{1, \dots, q\}$$

$q_0, q_{acc}, q_{rej}$

4 insist  $q_0 = \text{state } 1$

$q_{acc} = \text{" } 2$

$q_{rej} = \text{" } 3$

$$\Sigma = \{1, \dots, b\}$$

$$\Gamma = \{1, \dots, b, \underset{\uparrow}{b+1}, \dots, c\}$$

So

$V$  is described by  $\{0, 1, \dots, q\}^*$

In this way

$\langle G \rangle$ ,  $G$  is essentially general, but we require  $V$  special

$$\langle G \rangle \subset \{0, 1, \dots, q, \# \}^*$$

=  $M$ , Turing machine

$$\langle M \rangle = \text{description of } M = \langle Q, \Sigma, \Gamma, \delta, \dots \rangle$$

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$



$$\{1, \dots, a\} \quad \{1, \dots, c\}$$

$$\delta: (1, 1) \mapsto (, , L/R)$$

$$\delta: (1, 2) \mapsto (, , L/R)$$

⋮

$L=1$

$R=2$