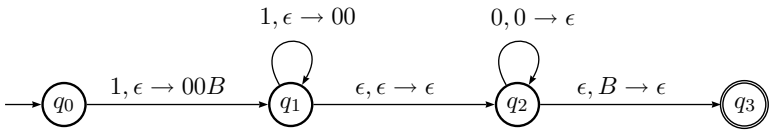
Midterm

Question 4

The grammar $S->\texttt{0}S\texttt{11}|\texttt{011}$ is an example of a CFG accepting L. An example of a pushdown automaton accepting L is:



(There are very many variants on this automoton.)

To show that L is not regular, assume that it is regular and use the pumping lemma. If p is L's pumping length, consider the word $w = \texttt{0}^p\texttt{1}^{2p}$. This word can be written as xyz with $|xy| \leq p$ and $xy^iz \in L$ for all non-negative integers i (and y is nonempty). But since w begins with p 0's, y must consist of a nonempty collection of 0's, and clearly $xz \notin L$ (contrary to the pumping lemma) since xz has $2p$ 1's but fewer than p 0's. Thus, by contradition, L cannot be regular. (This idea still works if you don't use the fact that $|xy| \leq p$, but then a few more cases arise.)