Marks
[10] 1. Give a brief explanation of how to take an NFA accepting a language, $L$, and form a new NFA accepting $L^{*}$. Same question for an NFA for $L_{1}$ and one for $L_{2}$, forming a new NFA for $L_{1} \cup L_{2}$. Explain why such constructions can be useful in taking a regular expression for a language and building a DFA accepting it.
[10] 2. Write a two state DFA for the language, $L$, of words over the alphabet $\Sigma=$ $\{0,1, \ldots, 9\}$ that in decimal represent integers divisible by 5 ; convert this to a regular expression using the general GNFA procedure. Is the regular expression you get the "smallest" or "simplest" regular expression for $L$ ?
[8] 3. Solve either part (a) or part (b), and explain your solution:
(a) Let $S$ be the set of all sets that don't contain themselves, i.e.,

$$
S=\{T \mid T \text { is a set with } T \notin T\} .
$$

Show that $S \in S$ leads to a contradiction. Show that $S \notin S$ leads to a contradiction. Do you think there can exist a "reasonable" set theory in which $S$ above exists (and is a set)?
(b) Let $L=\left\{a^{5}, a^{7}\right\}$. What is the minimum number of states needed in a DFA that recognizes $L$ ? $L^{*}$ ? Explain your answers, and if you use Myhill-Nerode, give the relevant equivalence classes. [Hint: 23 is the largest integer that cannot be written as non-negative multiples of 5 and 7 .]
[12] 4. Determine whether or not $L=\left\{0^{n} 1^{n} \mid n \geq 0\right\}$ is regular and whether or not it is context-free. Same problem for $L=\left\{0^{n} 1^{n} \mid n\right.$ is a power of 10$\}$.

# Be sure that this examination has 6 pages including this cover 

The University of British Columbia<br>Midterm Examinations - March 2007<br>Computer Science 421/501

Name $\qquad$

## Student Number

$\qquad$

## Instructor's Name

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## Section Number

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## Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. A sheet of notes will be provided.

## Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.
2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
(b) Speaking or communicating with other candidates.
(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
3. Smoking is not permitted during examinations.

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| 4 |  | 12 |
| Total |  | 40 |

