Marks

[8] **1.** Explain your answer to each question; if you answer "no," give a specific counterexample.

(a) If L_1, L_2 are regular languages, is $L_1 \cap L_2$ necessarily regular?
(b) If L_1, L_2 are context-free languages, is $L_1 \cap L_2$ necessarily context-free?
(c) If L_1 is a regular langauge and L_2 is a context-free language, is $L_1 \cap L_2$ necessarily regular?
(d) If L_1 is a regular langauge and L_2 is a context-free language, is $L_1 \cap L_2$ necessarily context-free? [Hint: thinking of DFA's and PDA's may help.]

Continued on page 3

[8] **2.** Explain your answer to each question; if you answer "no," give a specific counterexample. ("Acceptable" is the same as what the textbook calls "Turing-recognizable.")

(a) If L_1, L_2 are acceptable, is $L_1 \cup L_2$ necessarily acceptable?

(b) If L is acceptable, is the complement of L necessarily acceptable?

(c) If L is decidable, is the complement of L necessarily decidable?

(d) If L is acceptable but not decidable, is the complement of L necessarily not acceptable?

Continued on page 4

[5] **3.** Show that $\{a^n b^n c^n \mid n \geq 0\}$ is not context free.

Continued on page 5

[6] **4.** Recall that

$$A_{\text{TM}} = \{\langle M, w\rangle | M \text{ is a Turing machine and } M \text{ accepts } w\}.$$

Recall that we showed that A_{TM} is not decidable by the following argument (by contradiction). Suppose (to the contrary) that R is a Turing machine deciding A_{TM}. Let S a machine that on input $\langle M\rangle$ (with M a Turing machine) simulates R on input $\langle M, \langle M\rangle\rangle$ and outputs the opposite of R's answer. **Briefly** explain you answers to the following questions:

(a) Is it possible that S accepts on input $\langle S\rangle$?

(b) Is it possible that S rejects on input $\langle S\rangle$?

(c) Is it possible that S loops (i.e., does not halt) on input $\langle S\rangle$?

(d) In case S is given an input that is not of the form $\langle M\rangle$ with M a Turing machine, we did not specify what S should do. Does it matter?

Continued on page 6

[5] **5.** Let 5PLUS be the language of $\langle M \rangle$ such that M is a Turing machine that accepts at least five (distinct) strings. Show that 5PLUS is acceptable but not decidable. [You may assume that A_{TM} is not decidable.]

Continued on page 7

[5] **6.** In the following questions you may assume that SAT is NP-complete.

(a) Let DOUBLE-SAT be the set of $\langle\phi\rangle$ such that ϕ is a Boolean formula with at least two satisfying assignments. Show that SAT $\le_{\mathrm{P}}$ DOUBLE-SAT, i.e., reduce SAT to DOUBLE-SAT with a polynomial time reduction.

(b) Assume that you have done part (a). Show that DOUBLE-SAT is NP-complete. [I.e., what more needs to be said?]

Continued on page 8

[5] **7.** Briefly describe how to show 3SAT $\leq_P$ HAMPATH as indicated on the note sheet (this is the same reduction given in class and the text). Illustrate your reduction on the example $(x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3)$.

Continued on page 9

[6] **8.** Give short explanations to the following questions.

(a) Show that SAT $\leq_{\mathrm{P}} A_{\mathrm{TM}}$.

(b) Explain why part (a) does not imply that A_{TM} is NP-complete.

(c) Illustrate the reduction 3SAT $\leq_{\mathrm{P}}$ 3COLOR indicated on the note sheet (and done in homework) on the example $(x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3)$. [Recall that to "3 colour" a graph is to colour its vertices with three colours so that any two adjacent vertices are coloured differently.]

Continued on page 10

[5] **9.** Let L_n be the language of strings over $\{0, 1\}$ of length at least n whose n-th last digit is a 1. In other words, L_n contains precisely those strings of the form $u1w$, where w is a string of length $n - 1$ and u is an arbitrary (possibly empty) string.

(a) Write down an NFA accepting L_n with $O(n)$ states.

(b) Show that any DFA accepting L_n has at least 2^n states. [Hint: Show that there is a set of 2^n words that are pairwise distinguishable by L_n, i.e., such that any two of them x, y that are distinct have the property that for some z, exactly one of zx, zy is accepted by L_n.]

(c) What is the significance of the combination of parts (a) and (b)?

Continued on page 11

Continued on page 12

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Be sure that this examination has 12 pages including this cover

The University of British Columbia

Final Examinations - December 2007

Computer Science 421/501

Closed book examination — Time: 150 minutes

Name ____________________ **Signature** ____________________

Student Number ____________________ **Instructor's Name** ____________________

Section Number ____________________

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. Note sheets will be provided.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

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Total		53