

# 2004 Brief Solutions to Final, Selected Problems

Note Title

12/1/2007

1(a) Yes -  $L_1 \cap L_2 = \overline{(\overline{L_1} \cup \overline{L_2})}$ , and reg langs closed under  $\cup$  (from reg expressions), and complementation (by exchanging accepting and nonaccepting states of DFA).

1(b) No - let  $L_1$  be non-regular (e.g.  $\{0^n 1^n\}$ ) and  $L_2 = \overline{L_1}$ . Then  $L_1 \cup L_2 = \Sigma^*$  is regular, while  $L_1, L_2$  are not.

Sorry for the repeated (b)'s

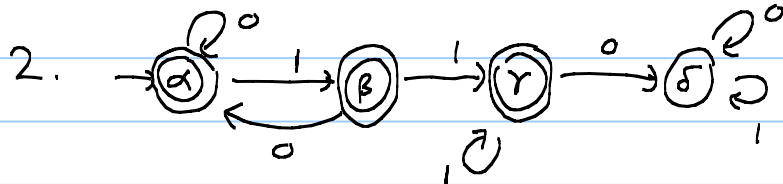
1(b) No - let  $L_1$  be  $\{0^n 1^n\}$ , and  $L_2 = 1^*$ .

1(b) Yes, just run the TMs for  $L_1$  and  $L_2$  simultaneously.

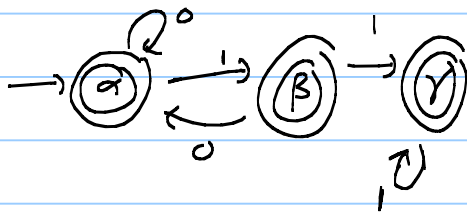
1(b) No -  $A_{TM}$  is acceptable, but not  $\overline{A_{TM}}$ .

1(c) Yes - just switch accepting and rejecting states.

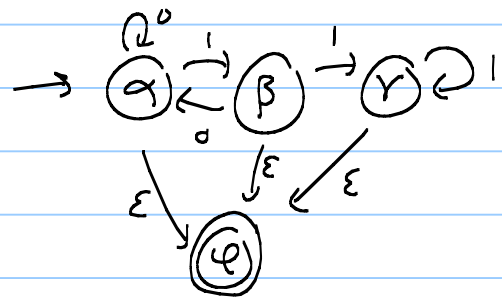
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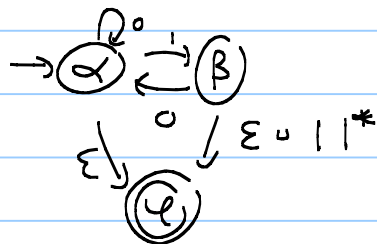
GNFA's: remove  $\delta$ :



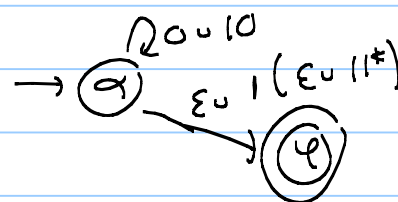
add unique final state



remove  $\gamma$ :



remove  $\beta$ :



reg exp:  $(0 \cup 10)^* (\epsilon \cup 1(\epsilon \cup 11^*))$ .

3. See text — pump  $0^p 1^p 0^p 1^p$

5. See class notes & text.

6. Construct a TM that runs in stages  $1, 2, 3, \dots$  such that on stage  $i$  we simulate  $M$  for  $i$  steps on all strings of length  $\leq i$ , and halts whenever  $M$  accepts  $S$  strings at any stage. This TM accepts SPLUS.

Given  $\langle M, w \rangle$ , let  $R =$  function of  $M, w$  be the TM that ignores its input and simulates  $M$  on  $w$  (and accepts when  $M$  accepts).

Then  $\langle M, w \rangle \in A_{TM} \Leftrightarrow R = R(M, w) \in \text{SPLUS}$ .

Hence  $A_{TM} \leq_m \text{SPLUS}$  so SPLUS is not decidable.

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7. See class notes, text, and/or note sheet for final 2007.

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8. Idea:  $3\text{SAT} \leq_p \text{DOUBLE-SAT}$  by adding an extra Boolean variable to the 3SAT instance, so the 3CNF is satisfiable iff DOUBLE-SAT has at least 2 satisfying assignments.

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9. See homework solutions.