CPSC 536F Nov 28, 2025

Last time:

A♠ > A♥ > ... > 2♦ > 2♣

card $i=52$ (A♠) card $i=1$ (2♣)

Card i:

Alice \ Betty	Call	Fold
Bet	μ_i	1
Fold	-1	-1

Payout to Alice
(from Betty)

Q: In what sense is "dual" of ridge regression actually a dual

Q: Why should you take Math 340 (Linear Programming)

(textbook Chvátal)

Ch 2, 3, 5, 10

teach the course

read the textbook

1.6.

Alice	Betty Call	Fold	Card_i
Bet	μ_i	1	
Fold	-1	-1	

μ_i = Expected gain for Alice:

Card i: $1, 2, \ldots, 52$

Cards less than i $\quad i-1$

$$\frac{i-1}{51} = \text{Prob Alice wins}$$

— 1.7 —

Prob ~~Betty~~ wins $= 1 - \frac{i-1}{51}$

$= \frac{52-i}{51}$

$\mu_i = 2\left(\frac{i-1}{51}\right) + (-2)\left(\frac{52-i}{51}\right)$

$= \frac{4i-106}{51}$

	C	F
B	$\frac{4i-106}{51}$	1
F	-1	-1

Card i

GAME 1

So for $i = 1, \ldots, 52$, you have to chose p_i : $p_1, p_2, \ldots, p_{52}$

You have to announce this to Betty.

$0 \le p_i \le 1$. So now Betty gets to choose ...

−1.9.

Value of game to Alice:

Alice \ Betty	C	F	
B	$\frac{4i-106}{51}$	1	$\leftarrow p_i$
F	-1	-1	$\leftarrow (1-p_i)$

Betty Call

$$\frac{1}{52}\sum_{i=1}^{52}\left(p_i\left(\frac{4i-106}{51}\right) + (1-p_i)(-1)\right)$$

Betty F

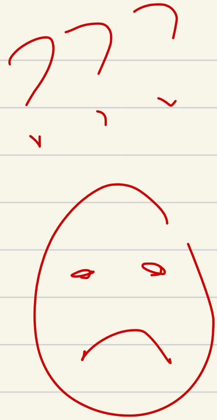
$$\frac{1}{52}\sum_{i=1}^{52}\left(p_i + (-1)(1-p_i)\right)$$

$$= \frac{1}{52}\sum(2p_i-1)$$

$i = 1, \ldots, 52$, each with prob $\frac{1}{52}$

Alice wants to choose

$p_1, \ldots, p_{52}$ s.t.



$$\operatorname{Min}\left(\underbrace{\sum_{i=1}^{52} p_i \left(\frac{4i-108}{51}\right) + p_i - 1}_{\ell_1(p_1,\ldots,p_{52})},\ \underbrace{\sum_{i=1}^{52} (2p_i - 1)}_{\ell_2(p_1,\ldots,p_{52})} \right)$$

is maximized, at some

$p_1^*, p_2^*, \ldots, p_{52}^*$ (optimal for Alice)

"Affine linear"

$a_1 p_1 + a_2 p_2 + \dots + a_{52} p_{52} + c$

Good news! Alice!

max μ

s.t.

$\mu \le \ell_1(p_1, \dots, p_{52}) + 3000$

$\mu \le \ell_2(p_1, \dots, p_{52}) + 3000$

"if needed"

54 total + 53 dec

$\mu, p_1, \dots, p_{52} \ge 0$

$p_1 \le 1$

$p_2 \le 1$

$\vdots$

$p_{52} \le 1$

— 1.9.3 —

vars 53 $\mu_1, p_1, \ldots, p_{52}$

ineq 54 = "# slack variables"

(Standard form LP (linear program))

$$\max \ \vec{c}^T \cdot \vec{x}_{dec}$$

s.t. $A\vec{x}_{dec} \leq \vec{b}$

$\vec{x}_{dec} \geq 0$

$\vec{x}_{slack} = \vec{b} - A\vec{x} \geq \vec{0}$

Amazing fact

-1.9.3

$\vec{X}_{dec} = \vec{X}$ decision variables

$\vec{X}_{slack} = \vec{X}$ slack variables

Start:

$\vec{X}_{slack} = \vec{b} - A\vec{X}_{dec}$

Theory of simplex method

Final tableau or dictionary

# of 0's in an optimal solution

amount $\vec{X}_{dec}, \vec{X}_{slack} \geq$ # of decision ≥ 53

— 1.9.4 —

of $\vec{X}_{slack}$ + $\vec{X}_{dec}$ in

optimal solution ≥ 53 are 0

$\Rightarrow$ $p_1, \ldots, p_{52}$ μ ← could be 0

$p_1 = 1 \Rightarrow p_1 \neq 0$, $\underbrace{\text{slack ass with } p_1}_{=0}$

$p_1 = 0 \Rightarrow p_1 = 0$

$0 < p_1 < 1 \Rightarrow$ both $p_1 \neq 0$
and
$p_1 < 1$

-1.9.5-

Theory alone $\Longrightarrow$

all but 1 of

$p_1, \ldots, p_{52}$ are 0 or 1 !!!

p_{52} Alice sees Ace of Spades

$= 1, 0$

$\vdots$

$p_1 \quad = 1, 0$

?

$$p_{52} \geq p_{51} \geq \ldots \geq p_1$$

$$1 \geq p_{s_2} \geq p_{s_1} \geq \cdots \geq p_{i_0+1}$$

$$\gg p_{i_0} \gg p_{i_0-1} \geq p_{i_0-2} \geq \cdots \geq p_1 = 0$$

Class Ends

Not done on Nov 28.

$$\mu_i = 2\,(\text{Prob Alice wins}) - 2\,(\text{Prob Betty wins})$$

These are just here to make sure computation earlier matches up..

$$= 2\left(\frac{i-1}{51}\right) - 2\left(\frac{52-i}{51}\right)$$

$$= \left(\frac{2}{51}\right)(2i-53)$$

Game 1: Alice picks $p_{52}, \ldots, p_1 \in [0,1]$ announces to Betty these values.

Betty wants to minimize between Call or Fold

Claim: Expected gain to Alice

is

Betty Calls	Betty Folds
$p_i \quad 2\left(\frac{2i-53}{51}\right)$	$p_i \quad (1)$
$+$	
$(1-p_i) \quad -1$	$(1-p_i)\,(-1)$
$\underbrace{\frac{1}{52}\sum_{i=1}^{52}\left(2p_i\left(\frac{2i-53}{51}\right) + p_i - 1\right)}_{\text{lin}_1(p_1,\dots,p_{52})}$	$\underbrace{\frac{1}{52}\sum_{i=1}^{52}\left(2p_i-1\right)}_{\text{lin}_2(p_1,\dots,p_{52})}$

Betty: Calls q, Folds $1-q$

Alice Bets $2p_i\left(\frac{2i-53}{51}\right)q + p_i(1-q)$

Ridge regression

$$\min_{w} \|Xw - y\|^2 + \lambda \|w\|^2$$

$$= \min_{Xw - y - z = 0} \|z\|^2 + \lambda \|w\|^2$$