CPSC 536F

Duality, and why you should take a course on linear programming...

= and read Chvatal's textbook Linear Programming

Poker:

Poker order:

A♠ > A♥ > A♦ > A♣

> K♠ > ... > 2♣

So 52 Cards, with a total ordering

Play the game in my Math 340 Course Handout, 2015-16

"Matrix Games and Poker"

The point: let $\mathcal{X}, \mathcal{Y}$ be sets, and

$F: \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$ a function

(in poker, $F(x,y)$ = payout to Player X, Player Y loses $-F(x,y)$, so "zero-sum")

$$m(F) \overset{\text{def}}{=} \max_{x \in \mathcal{X}} \left(\min_{y \in \mathcal{Y}} F(x,y) \right)$$

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[note "max" should be "sup", "min" $\to$ "inf"]

Lemma: $m(F) \leq M(F)$.

Def: $\mathrm{gap}(F) \overset{\text{def}}{=} M(F) - m(F)$ "duality gap"

<u>Abstract Theorem</u> (Stated conceptually):

① $$m(F) = \max_{x \in \mathcal{X}} \mathcal{F}_1(x), \quad \left(\begin{array}{c}\text{Player X} \\ \text{announces } x\end{array}\right)$$

where $$\mathcal{F}_1(x) = \min_{y \in \mathcal{Y}} F(x,y).$$

Say for each $x \in \mathcal{X}$ there is a $y_{opt}(x)$ s.t.

$$\min_{y \in \mathcal{Y}} F(x,y) = F(y_{opt}(x), x)$$

Say $\exists\, x^* \in \mathcal{X}$ s.t.

$$m(F) = \max_{x \in \mathcal{X}} \mathcal{F}_1(x) = \mathcal{F}_1(x^*)$$

② Similarly for $\mathcal{F}_y(y)$ and $x_{opt}(y)$.

and y^*.

If $\quad x_{opt}(y)$ unique for all y,

$\quad\quad y_{opt}(x)$ " " " x,

then

$gap(f) = 0 \implies$

$$y^* = y_{opt}(x^*)$$

and

$$x^* = x_{opt}(y^*) .$$

Game :

Alice, Player A

Betty, Player B

Part 1 : Alice & Betty ante

(put into the pot) 1 button

1 unit

Part 2 : 2 cards, from

52 : A spades, ... ~

1 : 2 clubs

are dealt (face down), one to

Alice and one to Betty, but
Alice looks at her card, not Betty's;
Betty look at no one's card.

Alice sees:

52, 51, __, 1

Part 3: Alice either folds

(then Betty gains 2 units
Alice " -2 units)

or bids 1 unit.

Part 4: If Alice has bid, then

Betty { folds
calls

Play the game a lot of times,

Alice & Betty have independent "source of randomness"

Alice bids on card i $\quad i = 1, \dots, 52$

with probability p_i.

$0 \leq p_i \leq 1$

Game #1 : Alice chooses

$p_1, \dots, p_{52}$, announces them

to Betty

Payout to Betty: for each $i \in [52]$

$\frac{1}{52}$ draws card i,

Alice bets on card i with prob

p_i.

Betty:

if Alice folds $\rightsquigarrow$ nothing to decide

if Alice bets $\rightarrow$ Betty calls

Betty folds

Say Alice draws card $52 = $ largest, ace of spades

Alice bets prob p_{52}

folds prob $1 - p_{52}$

$$\left.\begin{array}{l} p_{52} = 1 \\ \\ p_{52} < 1 \end{array}\right\} = \ldots ??$$

Alice has answered p_{52}