CPSC 536F Sept 8, 2025

- $\Phi(x^{test}, y^{test})$ vs.

$\mathrm{Avg}\left( \Phi(\text{cow}) \right)$

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Switch notation: $(x^{test}, y^{test})$ to

$\vec{x}^{\,test} = (x_1^{test}, x_2^{test})$

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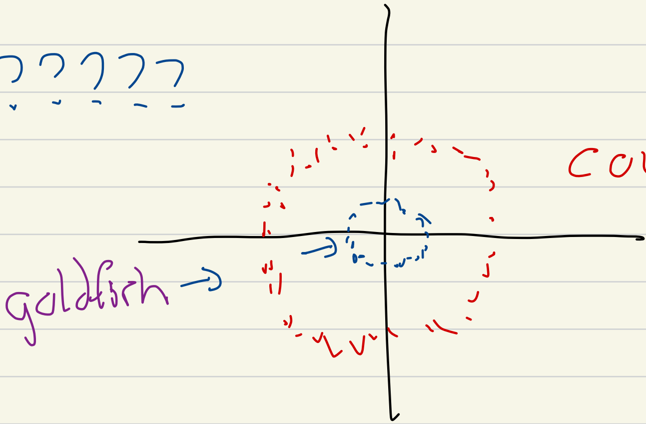
- The Fundamental Lemma:

$\vec{\varphi}^{\,test}$, $C_{avg} = \frac{1}{|C_i|} \sum_{c \in C_i} \vec{c}$,

$G_{avg} = \frac{1}{|G|} \sum_{g \in G} \vec{g}$

?????

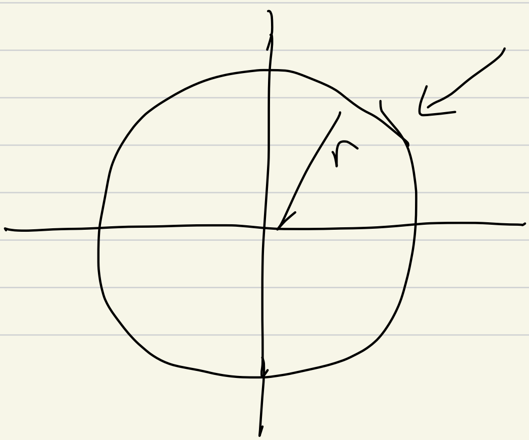



$\mathbb{R}^2$

No linear separator

(1) I declare "cows"

"lizards"




uniform distribution

$d\theta \ \frac{1}{2\pi}$

Avg Value on circle radius $r$ $\quad (1, x, y, x^2, xy, y^2)$

$$\downarrow$$

$$\left(1, 0, 0, \frac{r^2}{2}, 0, \frac{r^2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$\Phi : \mathbb{R}^2 \longrightarrow \mathbb{R}^6$ non-linear

$$(x, y) \longmapsto (1, x, y, x^2, xy, y^2)$$

$$\text{Avg Value}\left(\begin{matrix}\text{cows}\\ \text{in}\\ \mathbb{R}^2\end{matrix}\right) = (0, 0)$$

$\underset{\text{in } \mathbb{R}^2}{\text{AvgValue}}$ ( goldfish ) = $(0,0)$

$\mathbb{R}^2 \longrightarrow \mathbb{R}^6$

$\mathbb{R}^{\text{order}(n^2)}$

$\mathbb{R}^n \longrightarrow$ deg $\leq 2$ monomials

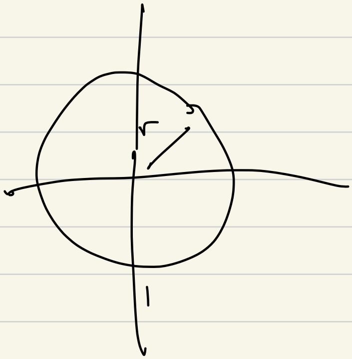
$\mathbb{R}^1 + \mathbb{R}^n + \mathbb{R}^{\binom{n+1}{2}}$

$\underline{x^2}, \underline{xy}, \underline{y^2}$

$\binom{n}{2}$ choose ~~2~~ diff

$n$ take $x_1^2, \ldots, x_n^2$

$$\binom{n}{2} + n = \binom{n+1}{2}$$



$\mathbb{R}^2$

$$(x,y) \mapsto (1, x, y, x^2, xy, y^2)$$

Avg Circle Rad $r$ $= \left(1, 0, 0, r^2/2, 0, r^2/2\right)$

$r_1$ = cows, $r_2$ = goldfish

$$\text{Avg } \Phi(\text{cows}) \approx \left(1, c, c, \frac{r_1^2}{2}, c, r_1^2/2\right)$$

$$\text{Avg } \Phi(\text{goldfish}) \approx \left(1, c, c, \frac{r_2^2}{2}, c, r_2^2/2\right)$$

$\mathbb{R}^6$

Imagine!

$$(x^{test}, y^{test}) \in \mathbb{R}^2$$

test data point

want! if $(x^{test})^2 + (y^{test})^2 = \rho^2$

Say: $\left(1, 0, 0, \frac{r^2}{2}, 0, \frac{r^2}{2}\right)$

$$\text{Avg}\left(\overline{\Phi}\begin{pmatrix}\text{circle} \\ \text{radr}\end{pmatrix}\right)$$

$$(x^{test}, y^{test}) \in \mathbb{R}^2$$

$$\overline{\Phi}(x^{test}, y^{test})$$

$$= \left(1, x^{test}, y^{test}, (x^{test})^2, x^{test}y^{test}, (y^{test})^2\right)$$

$$= (1, x, y, x^2, xy, y^2)$$

and

$$x^2 + y^2 = \rho^2$$

What is

fixed, test point

r's cows goldfish

$$\left\| \overbrace{(1, x, y, x^2, xy, y^2)}^{\text{fixed, test point}} - \left(1, 0, 0, \frac{r^2}{2}, 0, \frac{r^2}{2}\right) \right\|^2$$

$$= \left\| \left(0, x, y, x^2 - \frac{r^2}{2}, xy, y^2 - \frac{r^2}{2}\right) \right\|^2$$

$$= \underbrace{0^2 + x^2 + y^2 + \left(x^2 - \frac{r^2}{2}\right)^2 + (xy)^2 + \left(y^2 - \frac{r^2}{2}\right)^2}_{\text{Messy function}(r)}$$

Ignore

Messy function(r)

Strategy:

$$= \underbrace{f(x,y)}_{\text{indep of } r} + \underbrace{g(r,x,y)}_{\text{depend on } r}$$

$g(r, x, y)$

Reason: We only care about

MessyFunction$(r_1)$ − MessyFunction$(r_2)$

$r_1$ = radius of cow locations

$r_2$ = " " goldfish "

Ignore $c, x^2, y^2, (xy)^2$

Looking at

$$\underbrace{\left(x^2 - \frac{r^2}{2}\right)^2} + \left(y^2 - \frac{r^3}{2}\right)^2$$

$$\overbrace{x^4 - 2x^2\frac{r^3}{2} + \frac{r^4}{4}}$$

$$\overbrace{x^4 - x^2r^2 + \frac{r^4}{4}}$$

ignore $+ \; -y^2r^2 + \frac{r^4}{4}$

ignore

$$= \text{ignored} + \frac{r^4}{2} - (x^2+y^2)r^2$$

$$\frac{r^4}{2} - \rho^2 r^2$$

$$g(r, x, y) \rightsquigarrow g(r, \rho)$$

radius of test point

$\rho = \rho^{test}$

$$\underbrace{\frac{r^4}{2} - \rho^2 r^2}_{\text{minimize :}}$$

$x = \rho^2$

$a = r^2$

$$\frac{x^2}{2} - ax$$

$$\underbrace{\overset{f(x,y)+g(r_1,x,y)}{\text{MessyFunction}(r_1)}}_{\text{dist of } \Phi(x,y) \text{ to}} \overset{?}{>} \underbrace{\overset{f(x,y)+g(r_2,x,y)}{\text{MessyFunction}(r_2)}}$$

$$\text{Avg}\left(\Phi\left(\begin{matrix}\text{circle}\\ \text{radius}\\ r_1\end{matrix}\right)\right) \overset{?}{>}$$

$r_2$

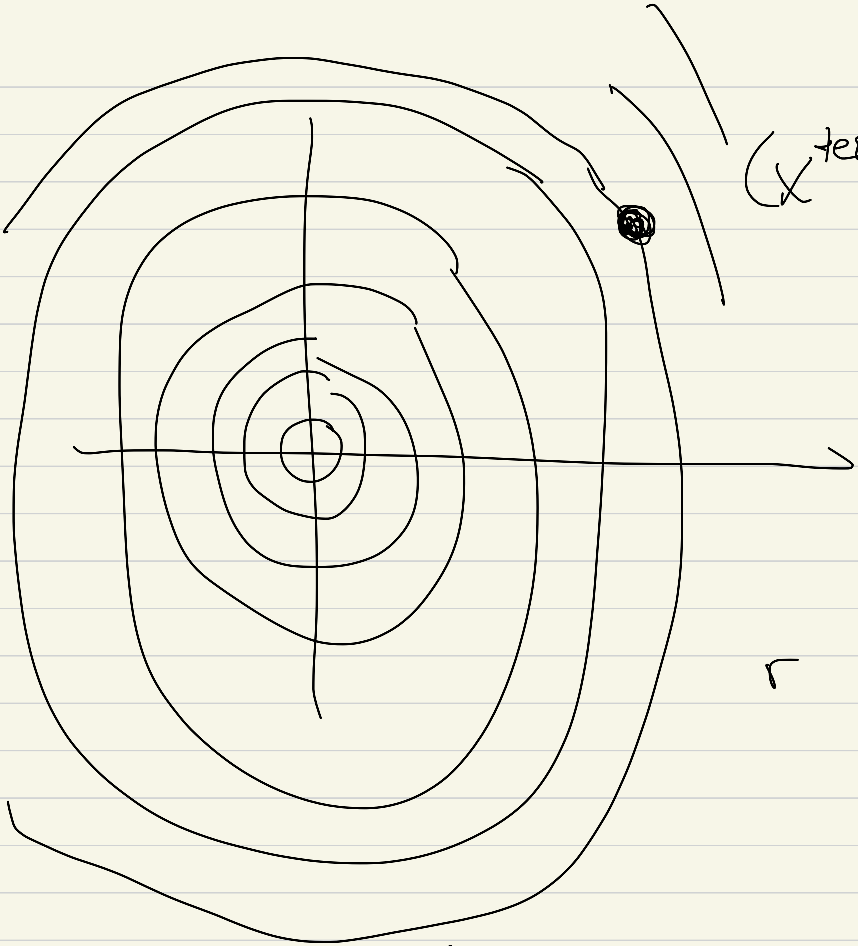
hopefully:

if $\rho = r_2$ then

$$>$$

and $\rho = r_1$

$$<$$




r variable

$$\mathrm{dist}\left( \Phi(x^{test}, y^{test}), \quad \mathrm{Avg}\ \Phi\begin{pmatrix}\text{Circle} \\ \text{Rad} \\ r\end{pmatrix} \right)$$