

CPSC 531F

April 7, 2025

Cancel office hours today.

Thursday: email me if you want to attend.

[Next few days will likely be noisier - location of hours to be moved.]

Today!

(1) Finish barcode decomposition

(2) Store at a few examples in

textbook:

(2020)

TDA in Genomics & Evolution

- Nice pictures, nice algorithms

Inductively:

We've done (C, q) -bars, $C \leq q \leq n$

whose sum is

$$V^C \rightarrow V^{C \rightarrow 1} \rightarrow V^{C \rightarrow 2} \rightarrow \dots \rightarrow V^{C \rightarrow n}$$

next step: build $(1, q)$ -bars for

$$1 \leq q \leq n$$

$$\bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \emptyset$$

$$\bar{V}^1$$

summing (C, q) -bars, $(1, q')$ -bars:

$$\bar{V}^C \rightarrow \bar{V}^1 \rightarrow \begin{array}{l} \text{part of } \bar{V}^2 \\ \text{containing} \\ \text{images} \\ \text{of } \bar{V}^C, \bar{V}^1 \text{ in } \bar{V}^2 \end{array}$$

enough to say $\left\{ \begin{array}{l} \text{Image } L^{0 \rightarrow 2} \text{ or } L^{0 \rightarrow 2} \bar{V}^0 \\ \textcircled{+} \text{ Image } L^{1 \rightarrow 2} \text{ or } L^{1 \rightarrow 2} \bar{V}^1 \end{array} \right.$

Image $L^{1 \rightarrow 2}$

$$= L^1(\bar{V}^1) = \bar{V}^{1 \rightarrow 2}$$

So need: enough $(1, q)$ - bars

to add to

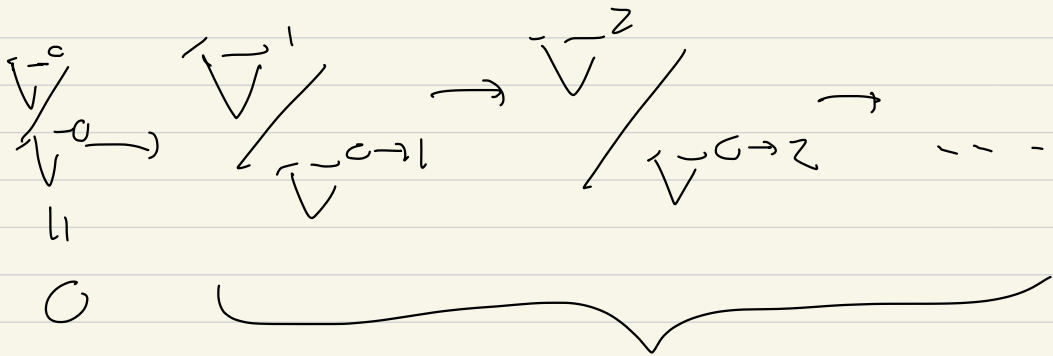
end of phase 0 $\left(\bar{V}^0 \rightarrow \bar{V}^{0 \rightarrow 1} \rightarrow \bar{V}^{0 \rightarrow 2} \rightarrow \dots \right)$
to get $\left(\bar{V}^0 \rightarrow \bar{V}^1 \rightarrow \bar{V}^{1 \rightarrow 2} \rightarrow \bar{V}^{1 \rightarrow 3} \rightarrow \dots \right)$

end of phase 1

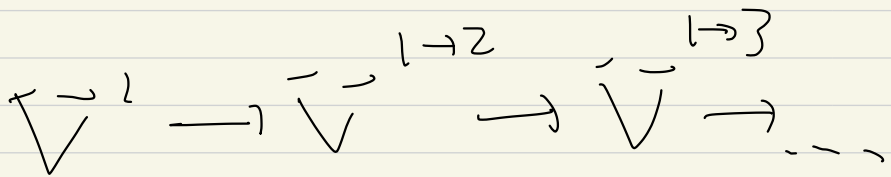
METHOD 1

Could think "more canonically"

as



then "lift" every bar to



METHOD 2:

Perform something similar for the $(1, q)$ -bars that we did for $(0, q)$ -bars

Then we'll have

$$\bar{V}^0 \rightarrow \bar{V}^1 \rightarrow \bar{V}^{1 \rightarrow 2} \rightarrow \bar{V}^{1 \rightarrow 3} \rightarrow \dots$$

next phase: add $(0,2)$ -bars:

end of phase 1

$$\bar{V}^0 \rightarrow \bar{V}^1 \rightarrow \bar{V}^{1 \rightarrow 2} \rightarrow \bar{V}^{1 \rightarrow 3} \rightarrow \dots$$

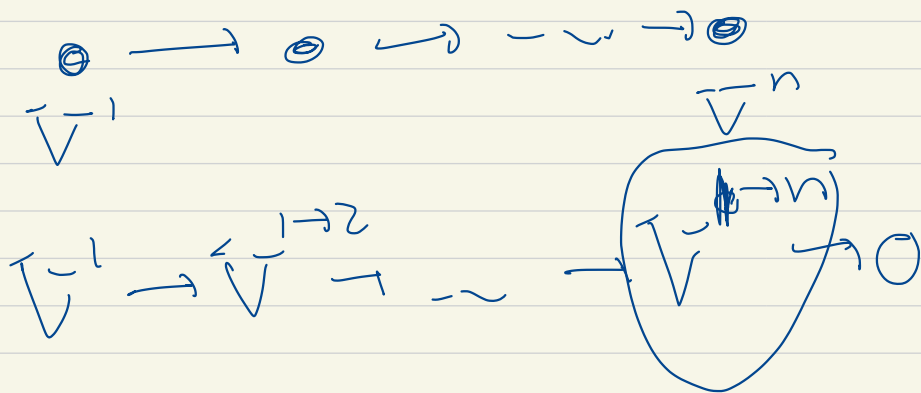
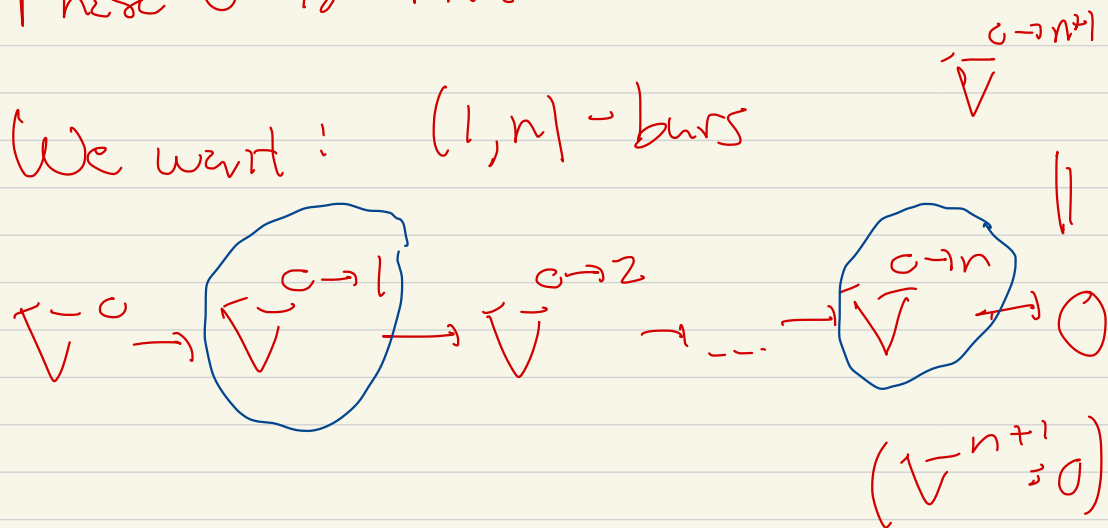
Phase 2

add $(2,1)$ -bars

to get

$$\bar{V}^0 \rightarrow \bar{V}^1 \rightarrow \bar{V}^2 \rightarrow \bar{V}^{2 \rightarrow 3} \rightarrow \dots$$

Phase 0 to Phase 1



We pick a basis for

$$V^{-1 \rightarrow n} / V^{0 \rightarrow n}$$

Definition:

Given $\bar{U} \subset \bar{W}$ vector spaces:

a basis of \bar{W} relative to \bar{U} :

w_1, \dots, w_r , $r = \dim(\bar{W}) - \dim(\bar{U})$

s.t.

① Every vector in \bar{W} can be written uniquely as a linear combination of w_1, \dots, w_r plus some vector in \bar{U}

② $w_1 + \bar{U}, w_2 + \bar{U}, \dots, w_r + \bar{U} \in \bar{W}/\bar{U}$

these r -vectors in quotient \bar{W}/\bar{U} are a basis

(3) If u_1, \dots, u_m are any basis of \bar{U} , then

$$u_1, \dots, u_m, w_1, \dots, w_r$$

are a basis of \bar{W}

(4) \bar{W} is an internal direct sum,
~~or~~ direct sum of subspaces

$$\text{Span}(w_1), \text{Span}(w_2), \dots, \text{Span}(w_r), \bar{U}$$

(4') \bar{W} is \dots of

$$\text{Span}(w_1, w_2, \dots, w_r), \bar{U}$$

(4') (5) (6) (7) u_1, \dots, u_m represent
 "lifting" a basis of \bar{W}/\bar{U}

So pick a relative basis

$w_1^{1 \rightarrow n}, \dots, w_s^{1 \rightarrow n}$ of $\bar{V}^{1 \rightarrow n}$
relative to $V^{0 \rightarrow n}$

$$V^{0 \rightarrow n} \subset \bar{V}^{1 \rightarrow n}$$

$w_1^{1 \rightarrow n}, \dots, w_s^{1 \rightarrow n} \in \bar{V}^{1 \rightarrow n}$

so $\uparrow \mathcal{L}^{1 \rightarrow n}$ $\uparrow \mathcal{L}^{1 \rightarrow n}$

$v_1^{1 \rightarrow n}, \dots, v_s^{1 \rightarrow n}$ \bar{V}^1

So we have

$$V_1^{0 \rightarrow n}, V_2^{0 \rightarrow n}, \dots, V_{m_{0,n}}^{0 \rightarrow n}$$

$$m_{0,n} = \dim(V^{0 \rightarrow n})$$

$$V_1^{1 \rightarrow n}, \dots, V_5^{1 \rightarrow n} \text{ in } V^{1 \rightarrow n}$$

So that

$$\left\{ \mathcal{L}^{0 \rightarrow n} \left(V_i^{0 \rightarrow n} \right) \right\}_{i \in [m_{0,n}]} \cup \left. \right\} \mathcal{B}$$

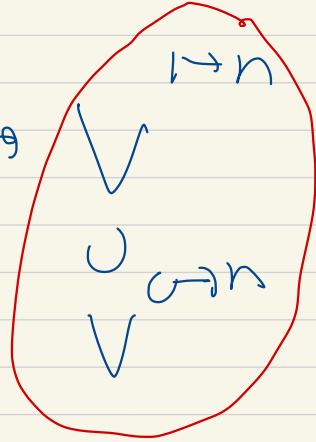
$$\left\{ \mathcal{L}^{1 \rightarrow n} \left(V_i^{1 \rightarrow n} \right) \right\}_{i \in [5]}$$

we get a basis for $V^{1 \rightarrow n}$

V^n

U

basis B

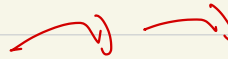


$$\left\{ V_i \right\} \xrightarrow{\quad} \left\{ \begin{matrix} 0 \rightarrow 1 & 0 \rightarrow n \\ L & V_i \end{matrix} \right\}$$

$$\left[V_i \right]$$

V^0

V^1



V^n

basis

Claim: any $1 \leq j \leq n$

$$\mathcal{L}^{0 \rightarrow j} \left\{ V_i^{0 \rightarrow n} \right\}$$

$$\mathcal{L}^{1 \rightarrow j} \left\{ V_i^{1 \rightarrow n} \right\}$$

are a basis for $V^{1 \rightarrow j}$.

These are linearly independent in $V^{1 \rightarrow j}$
and claim:

$$\# \text{ vectors} = \dim \left(V^{1 \rightarrow n} \right)$$

!