CPSC 421/501 Nov 28, 2025 [Sip]

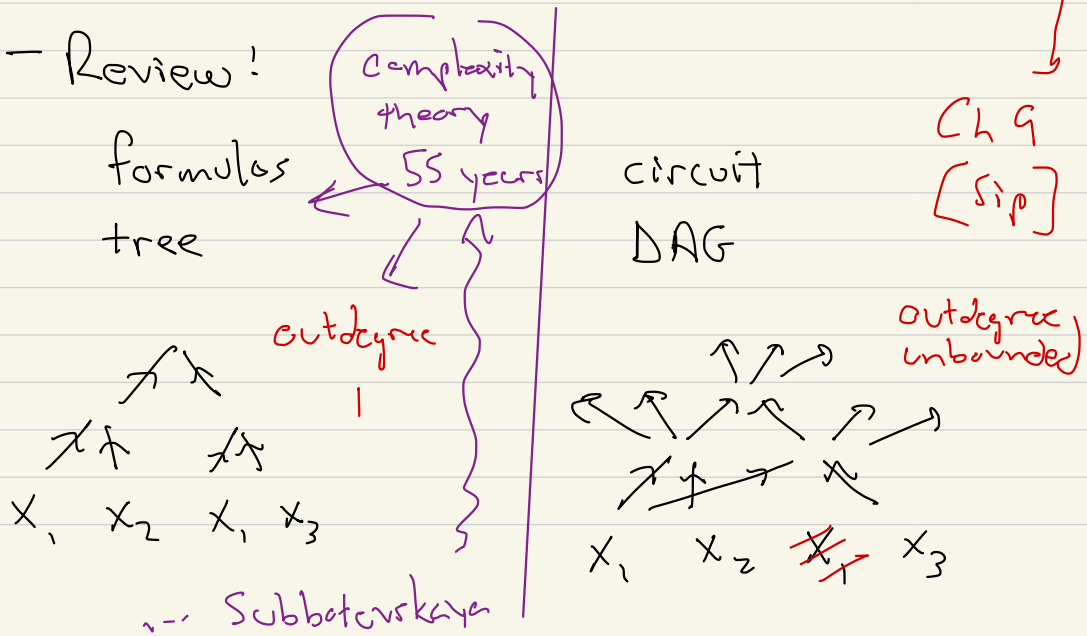
Ch 7.

- Cook-Levin for non-deterministic T.m.s: $L \in NP \Rightarrow L \leq_p 3SAT$

- Cook-Levin idea for deterministic T.m.s: $L \in P \Rightarrow L$ has polynomial circuits (Using same x_{ijr}, y_{ij}, z_{iq})

Ch 9 [Sip]

- Review!

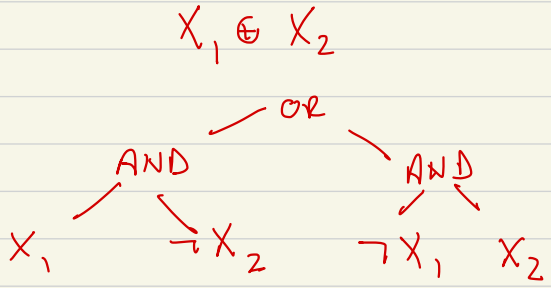


Subbotovskaya (1961): [Random restrictions]

Let f be a formula for $x_1 \oplus x_2 \oplus \ldots \oplus x_n$. Then

$$\text{Size}(f) \geq n^{1.5}$$

Remark:



Recurse:

$x_1 \oplus \ldots \oplus x_{2^k}$ has size $(2^k)^2$ formula

Corollary: For Andre'ev function on n-variables, a formula requires $\geq C n^{2.5}$ size

(and $O(n^3)$ formula is easy to produce)

Admin:

HW 9 is last homework to be collected.

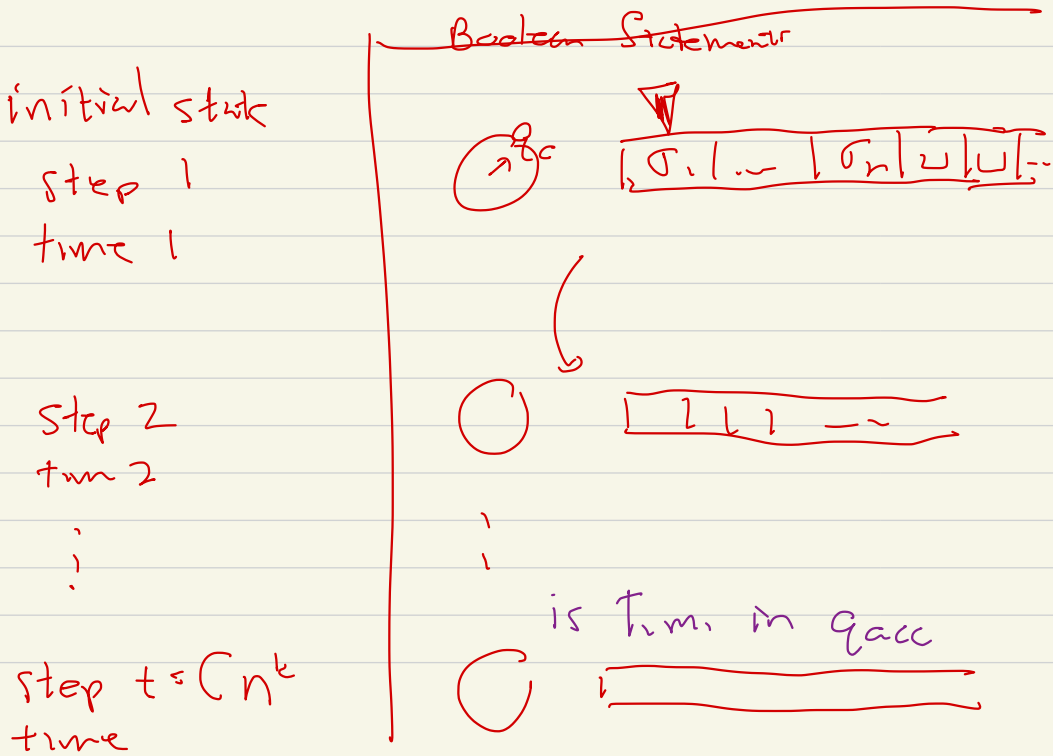
↝ HW 10 + final exam review problems

Final: Friday, Dec 15, 8:30am

You have a T.m. $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

non-det $\delta : Q \times \Gamma \to \text{Power}(Q \times \Gamma \times \{L, R\})$

det $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$



$X_{ijγ}$ = symbol γ at time i ? cell j

Y_{ij} = tape head in cell j time/step i

Z_{iq} = are we in state q at time/step i

Step 1:

Cook-Levin Thm:

- Are we in state q_0?
- Is tape head over cell 1?
- What is written on tape

Boolean variables i = 1

Step 2

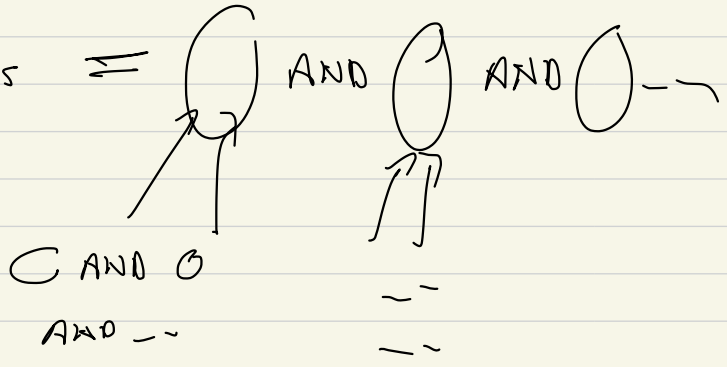
- Write same configuration --

Is this an allowed transition for our Turing

Boolean vars i = 2

Cook-Levin Construction:

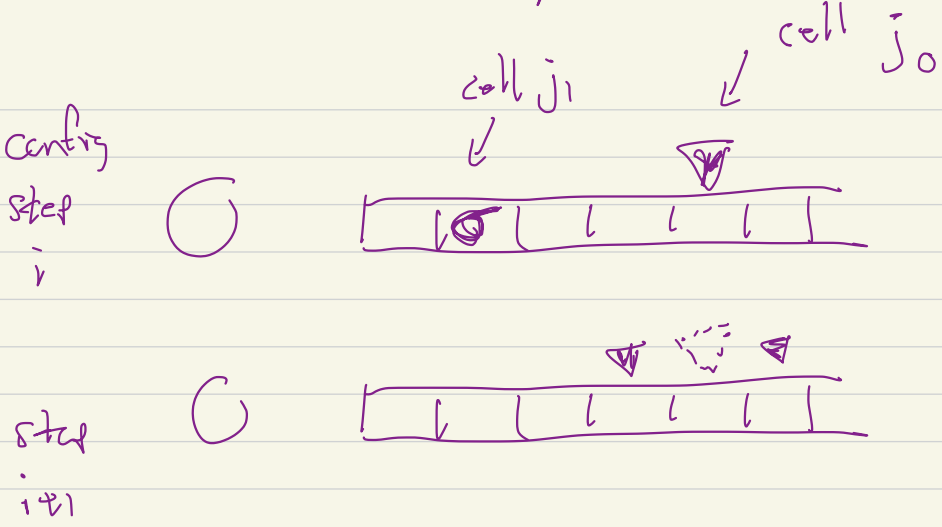
Non-deterministic:

all conditions = 

we have x_{ijr}, y_{ij}, z_{iq} but can add new variables, get

formula $f = f(M; \underbrace{\sigma_1, \ldots, \sigma_n}_{\text{input}})$

Example: Is state/configuration in step/time $t+1$ allowable from step t?

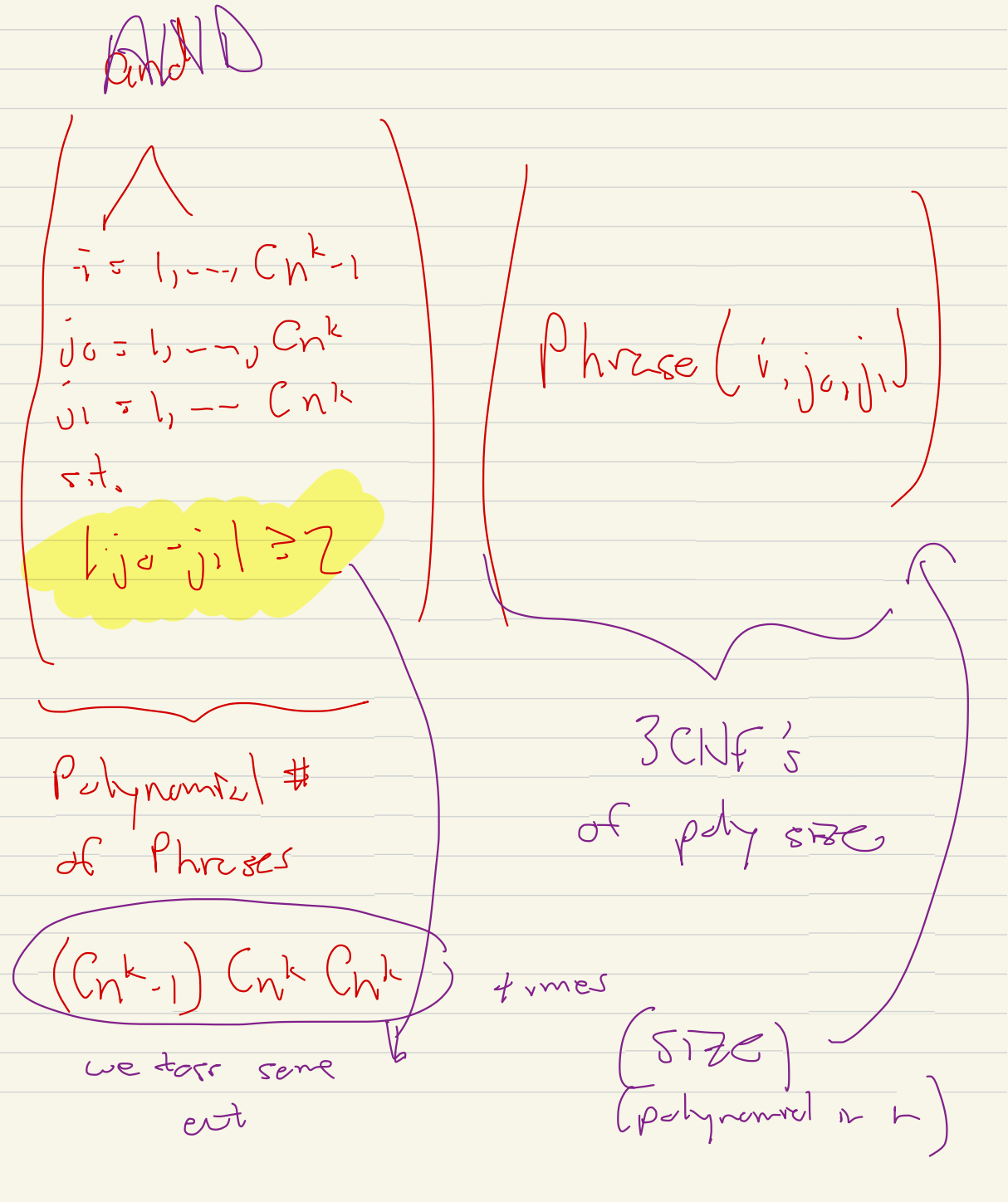


Say: $|j_0 - j_1| \geq 2$

if tape head is at j_0 at time i
and $|j_1 - j_0| \geq 2$ then

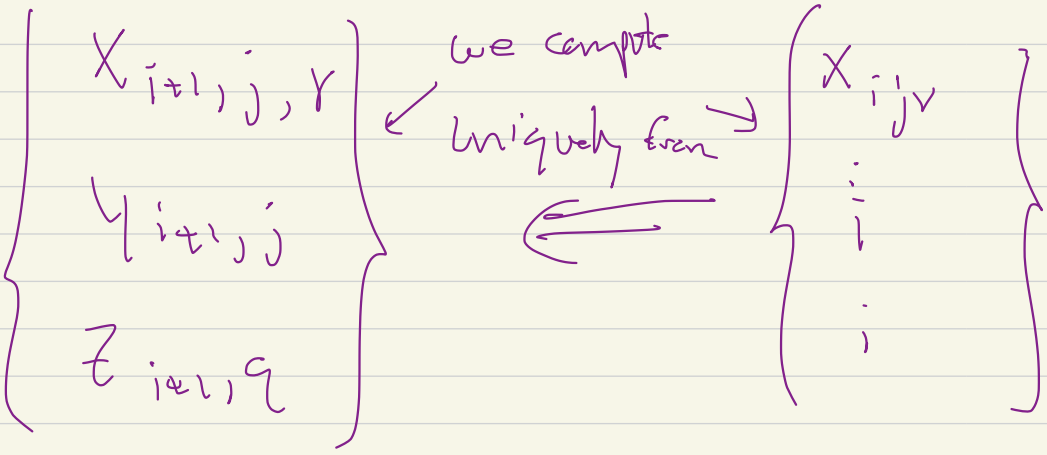
time $i+1$: cell entry at j_1
is same as " " " time i

Phase (i, j_0, j_1)


AND
$i = 1, \ldots, Cn^k - 1$
$j_0 = 1, \ldots, Cn^k$
$j_1 = 1, \ldots Cn^k$
s.t.
$|j_0 - j_1| \geq 2$
Phrase (i, j_0, j_1)
Polynomial # of Phrases
$(Cn^k - 1)\, Cn^k\, Cn^k$
4 times
we don't care about
3CNF's of poly size
(SIZE)
(polynomial in n)

If M is deterministic:

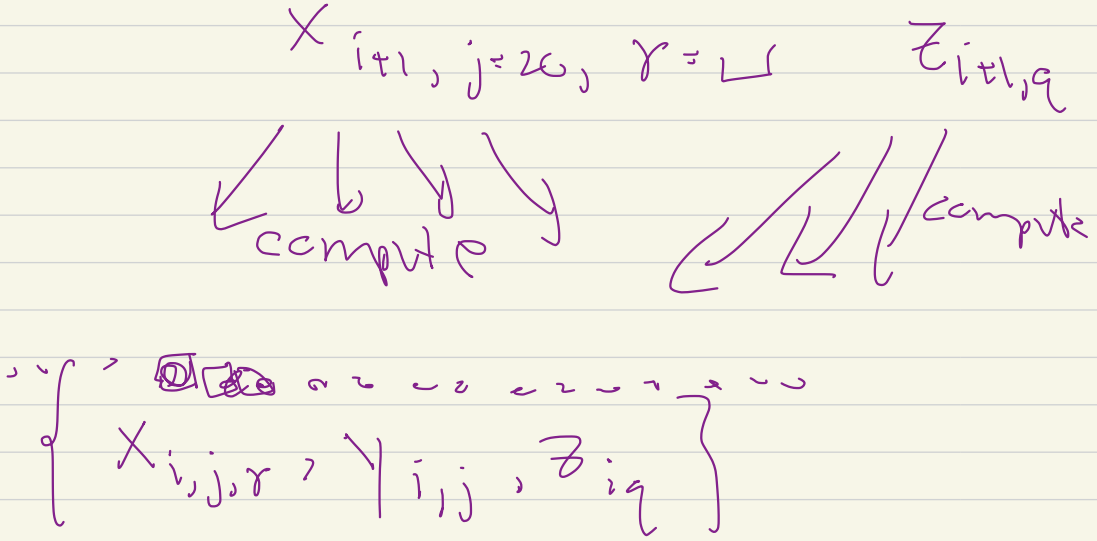
at step $i+1$, there is a unique way to get there from step i



$j \in [Cn^k]$

$\gamma \in \Gamma$

$q \in Q$



deterministically, if M is deterministic

M non-deterministic

Phase(i, j_0, j_p) = SimplerPhrase(i, j_0, j_1)

(if tape head is at j_0 at time i
and ~~$t_{j_1, i}, t_{j_0, i}$~~ then

time $i+1$: tape symbol at j_1
is same as ... time i)

AND $_{\gamma \in \Gamma}$ (if $y_{i,j_0} = T$ then

$x_{i,j_1,\gamma} = x_{i+1,j_1,\gamma}$)

↓

if $p = T$ then $q = r$

Boolean alg. p, q, r

$q = T$ and $r = T$

or

$q = F$ and $r = F$

$= f(p, q, r)$

So done by homework

(not getting into the Bool alg)