CPSC 421/501 Nov 19, 2025

- 2COLOUR & 3COLOUR

  - $\langle G \rangle$ where G is a graph

[Sip] Ch 3 Ch 7

$\langle \ \rangle$ = description of a graph

- 2COLOUR $\in P$, 3COLOUR $\in \underbrace{NP}_{\text{non-deterministic}}$

- Cook-Levin Theorem:

[Ch 7]

SAT $\in P \iff P = NP$

3SAT $\in P \iff$ "

3COLOUR $\in P \iff$ "

etc.

[Ch. 9]

Scholium to Cook-Levin:

$P = NP \Rightarrow \exists$ poly size circuits

for SAT (for 3SAT,

for 3COLOUR, etc.)

"almost"

$\Longleftarrow$

$\|$

Open: Does SAT (3SAT, 3COLOUR, etc.)

require $\geq n^{3.00001}$ size formulas?

$\|$

XOR requires $\geq n^{1.5}$ size formulas,

ANDRE'EV " $\geq n^{2.5}$ " "

Subbotovskaya (1961) restriction method

Admin! Joel's office hours moved to 12:25 - 1:40 pm tomorrow (Nov 20).

There are countably infinitely many

$$\left\{ \begin{array}{l} \text{DFAs} \\ \text{Turing machines} \end{array} \right\}$$

What is $\text{ACCEPTANCE}_{\text{Tm}}$

we needed to "standardize"

" " " describe

DFA's

T.m.'s

⋮

Graphs

$$\text{T.m.}: \ (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej}) \ \ldots$$

Standard: $Q = \{1, \ldots, n_Q\}$, $n_Q \in \mathbb{N}$, etc.

Simple graph: $G=(V,E)$,

V any set ← vertex set

E, $\subset$ all pairs of vertices

↑

edge set

"Standardize" or "Describe" graphs:

V must be $\{1,2,\ldots,n=n_V\}$

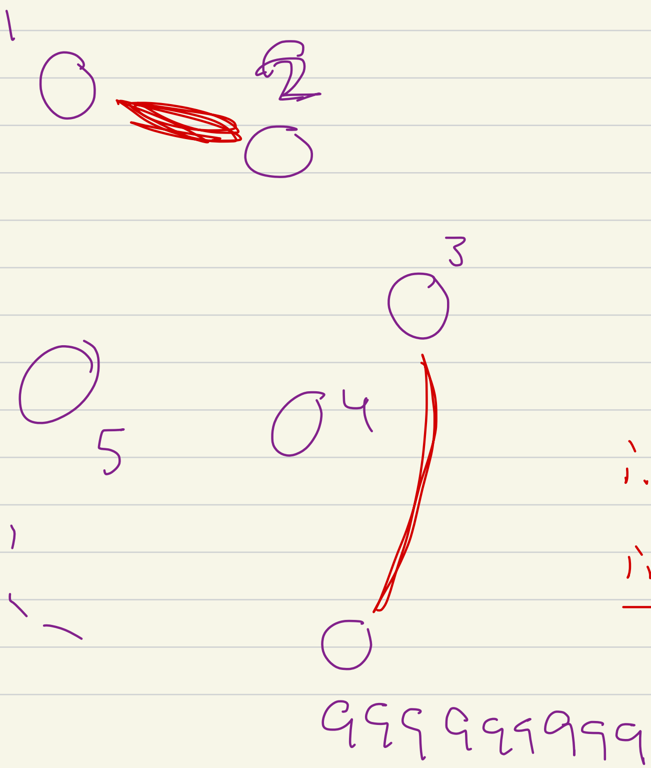
E is a subset of

$$\binom{V}{2}=\binom{[n]}{2}=\{\{1,2\},\{1,3\},\ldots,\{n-1,n\}\}$$

Rem! Example 1

$V = [10^{10}] = \{1, 2, \ldots, 10^{10}\}$

$E = \{\{1,2\}, \{3, 9999999999\}\}$



and

4, 5, ...

are <u>isolated</u>

i.e. have no

<u>incident</u> edges

$\langle G \rangle$ = write down # vertices,

describe one edge,

then " another edge,

⋮

Example 1: G as in example 1:

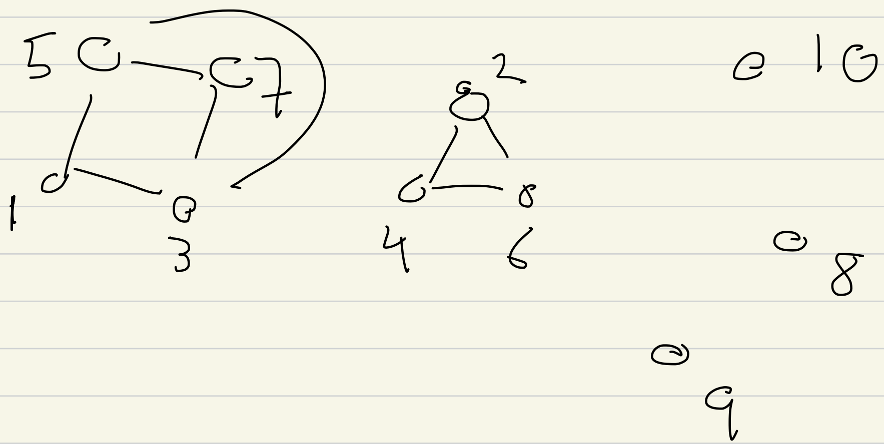
$\langle G \rangle$ =

Convention, as per class discussion,

we list all non-isolated vertices

first

Example 2:



{1,3}, {3,7},
{7,5} ---

|V|
~
10 # 1 # 3
3 # 7
2 # 4

|V| in
base 10
edges described
edge_1 description
edge_2 "

Edge {i, j} described
i base 10 # j base 10

$|$ and isolated vertices last

$$\text{2COLOUR} = \left\{ \begin{matrix} \langle G \rangle \\ \| \\ \langle V, E \rangle \end{matrix} \;\middle|\; \begin{array}{l} G \text{ is 2-colourable:} \\ \text{i.e. } \exists \text{ map} \\ f : V \to \{1, 2\} \\ \qquad\qquad \{\;,\;\} \\ \text{s.t. each edge} \\ \{i, j\} \in E \text{ has} \\ f(i) \neq f(j) \end{array} \right\}$$

$$\text{GRAPHS\_WITH\_ISOLATED\_VERTICES} = \left\{ \langle G \rangle \;\middle|\; \begin{array}{l} G \text{ has an} \\ \text{isolated vertex} \end{array} \right\}$$

3COLOUR → NP-complete

$= \{ \langle G \rangle \mid \ldots$

HAMILTONIAN CYCLES $= \{ \langle G \rangle \mid \ldots \}$

CLIQUE_SIZE_FIVE $= \{ \langle G \rangle \mid \ldots \}$

⋮

$O(n^5)$ time

on a RAM

$O(n^{10})$ on a

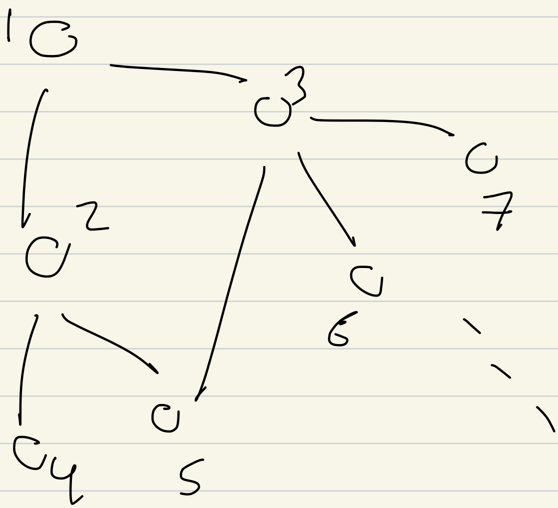
1-tape

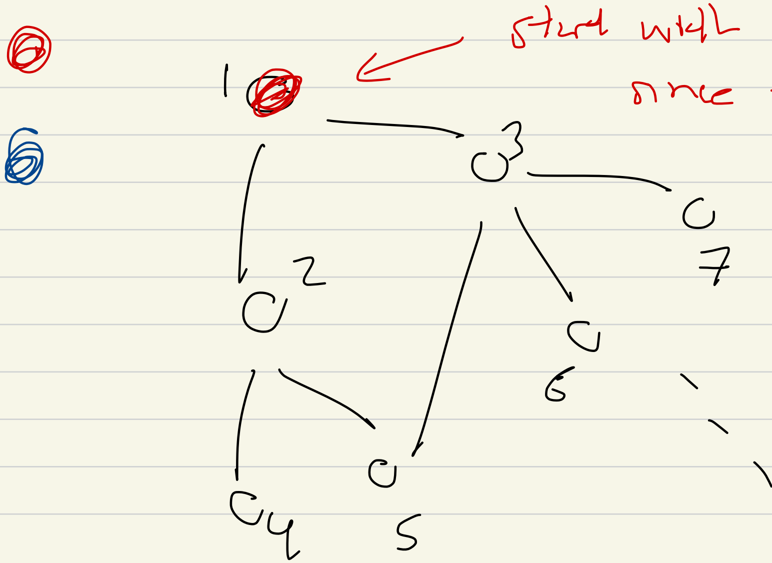
T.m.

all languages over

$$\Sigma_{\text{graphs}} = \{ 0, 1, \ldots, 9, \# \}$$

2 COLOUR! Is it 2 colourable?





start with red,
since there's a
symmetry...