CPSC 421/501 Nov 17, 2025

- P vs. NP Ch. 7 [NP-completeness]
  - $\Theta(f(n))$ [$O(f(n))$ - a joke]
  - $TIME(f(n))$, $NTIME(f(n))$

deterministic vs. non-deterministic time

Ch. 3

- $P = \bigcup_{k \in \mathbb{N}} TIME(n^k)$
- $NP = \bigcup_{k \in \mathbb{N}} NTIME(n^k)$

e.g. $2COLOR \in P$, $3COLOR \in NP$

- P=NP iff $3COLOR \in P$ (Cook-Levin)

↳ 3SAT

Admin! Next homework will be assigned Thursday, November 20, due November 27

CPSC 501: Please email:

To: jf@cs.ubc.ca

Subject: ... 501 ...

About your essay topic. Can work in groups of size 1 to 3.

$O(f(n))$ means ---

$n \in \mathbb{N} = \{1, 2, 3, \ldots\}$

$O(f(n))$ for $n \to \infty$,

$f : \mathbb{N} \to \mathbb{R}$

We define

$$O(f(n)) = \{ g : \mathbb{N} \to \mathbb{R} \text{ s.t.}$$

for some $C = C(g)$, (where C can depend on g, but not on n) we have

$$|g(n)| \leq C f(n) \text{ for } n \text{ sufficiently large, i.e. } n \geq n_0 \text{ some } n_0 \}$$

$$O(n^4) = \{ g : \mathbb{N} \to \mathbb{R} \text{ s.t. } \exists C, n_0 \text{ s.t. } \forall n \geq n_0, |g(n)| \leq C n^4 \}$$

contains!

$$n \longmapsto n^3 + 2n^2 + 1$$

$$n \longmapsto 20n^4 + 3(\sin(n))^3 \cdot n^3$$

$$n \longmapsto n^4(\log_2 n) \text{ is not in } O(n^4)$$

("of order n^4")

Wenning !

$$n \longmapsto n^4 + 2n^3 + n \cdot 10^{10^{10^{10}}}$$

$$\in O(n^4)$$

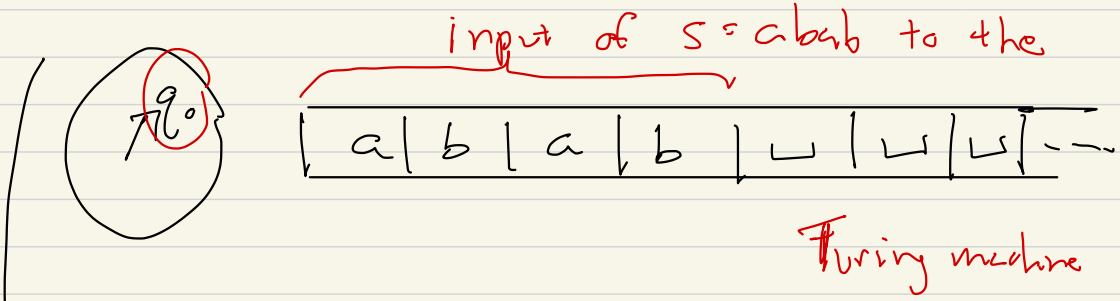
"uh. duh" of n^4 (Udi Manber)

$$OO(n^4)$$

$$TIME(f(n)) = \boxed{D}TIME(f(n))$$

deterministic Turing machines

1st step, initial configuration



n = input size = $|s|$ here 4

2nd step

3rd step

...

step t halts: 

The algorithm halts at step t if at step t, the state is either q_{acc} or q_{rej}, and steps $1, 2, \ldots, t-1$ the state isn't q_{acc} or q_{rej}.

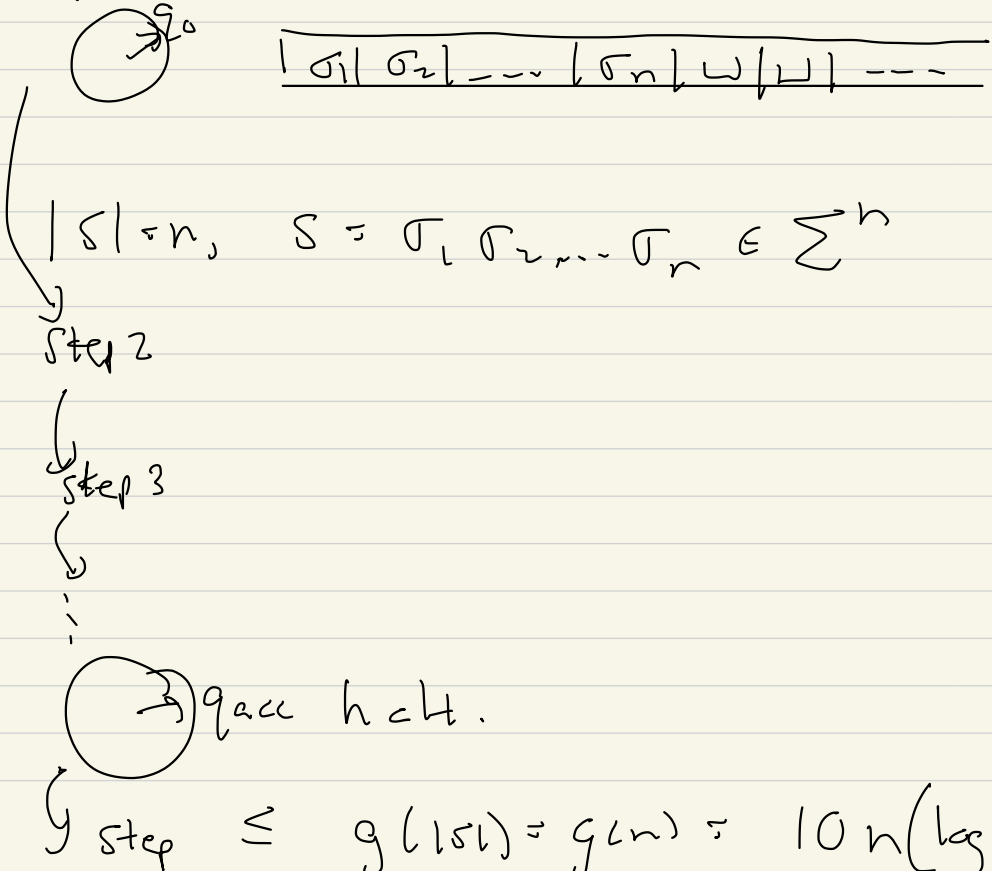
Equivalently, the algorithm, on input s takes time t.

We say that an algorithm (Turing machine) runs in time $g(n)$ if on any input, $s \in \Sigma^*$, the machine halts after at most $g(|s|)$ steps.

So $g(n) = 10n(\log_2 n)$,

then Tm. $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

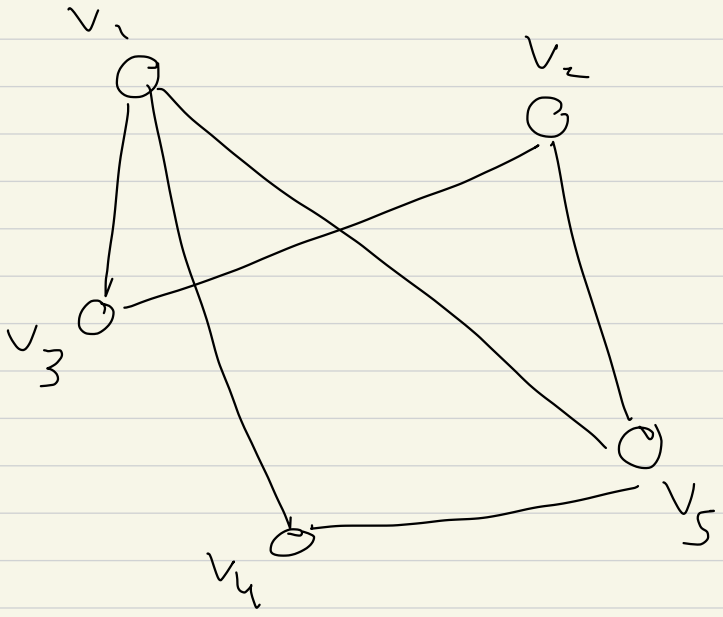
runs in time $g(n)$ if $\forall s \in \Sigma^*$ inputs



$t_{step} \leq g(|s|) = g(n) = 10n(\log_2(n))$

2 COLOUR :

Given a simple graph $G = (V, E)$



Formally :

V is a set, E = a subset of pairs of vertices

Example:

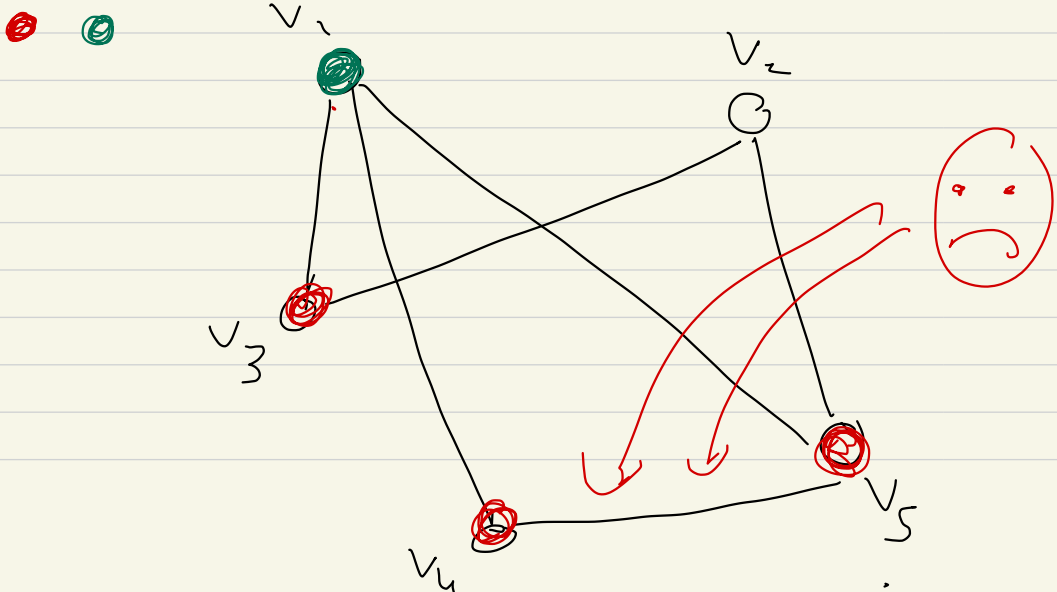
$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

$$E = \{ \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}$$

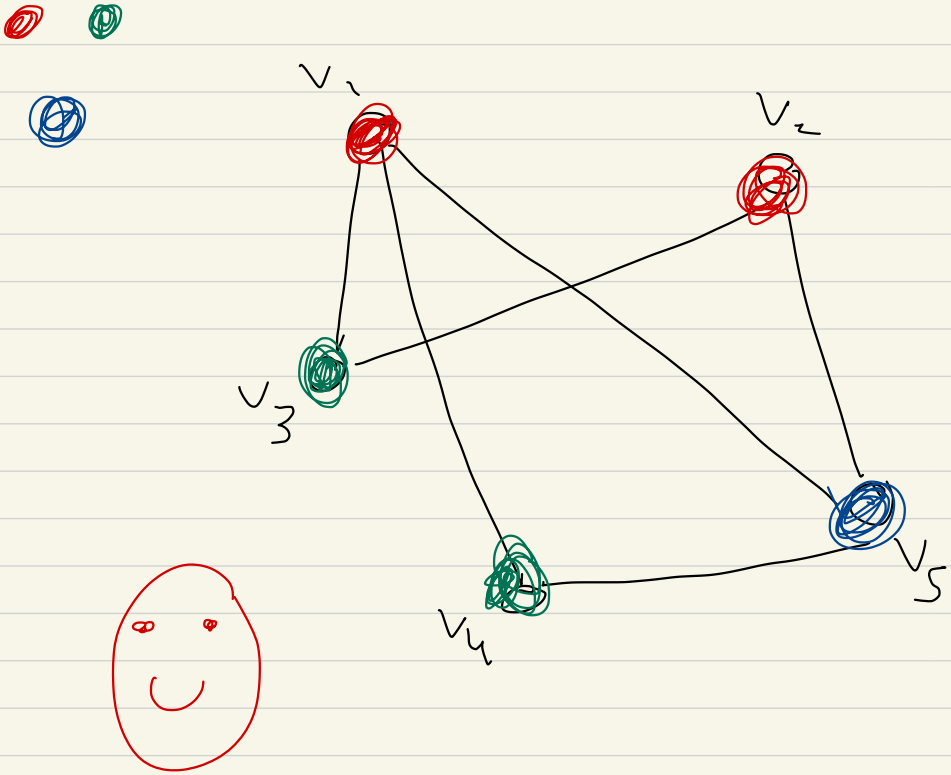
$$\{v_2, v_3\}, \{v_2, v_5\}$$

$$\{v_4, v_5\} \}$$

2 colour



3 COLOUR



3 COLOUR if you can solve it

$\Rightarrow$ 10^6 USD