CPSC 421/501 Nov 7, 2025

Today:

- Finish Phase 1 of 2-tape $\text{PALINDROME}_{\{a,b\}}$

- Thm: Any L decided by k-tape T.m. can be decided by 1-tape. Note! first t steps of a k-tape algorithm can simulated by $\leq Ct^2$ steps of a 1-tape machine

This $t \to Ct^2$ tradeoff "fight"
Proof: "Fooling methods"

Last part of class:

Any review questions for midterm?

To get a realistic idea of how difficult midterm vs. homework, look at "midterm review package"

$\rightsquigarrow$ previous years

1 High-level description

2 Implement, but q_1 = describe!

q_1 means the right moving part of Phase 1

3 Write down δ function values

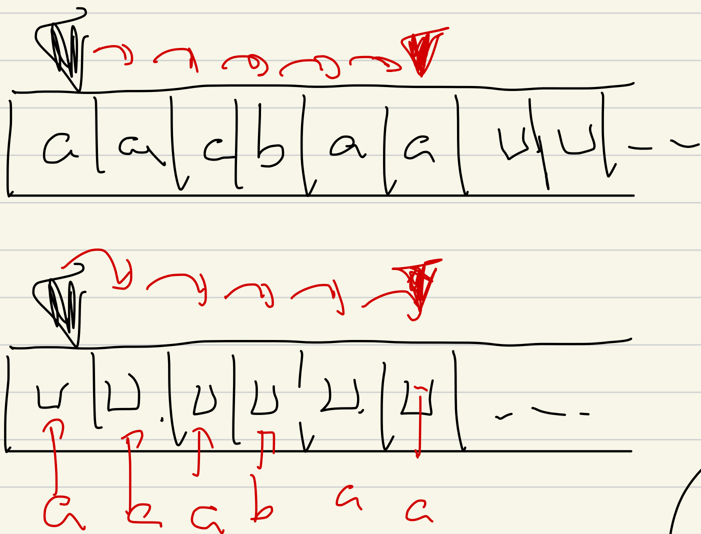
$\text{PALINDROME}_{\{a,b\}}$ 2-tape!

linear $\underbrace{\text{time}}_{\#\text{ steps}}$ algorithm

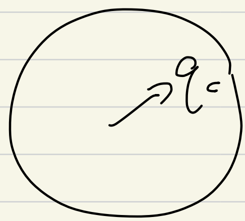
Ch 7

Alg:

Phase 1:



initial state
(Step 1)



Input on tape 1
Tape 2 is blank
→ ① Move both tape heads to the last cell # of which the input is written

② leave tape 1 cells alone
tape 2 cells copy

First mark both left edges

$$\Sigma = \{a, b\}, \quad \Gamma = \{a, b, \sqcup, a', b'\}$$

First move:

$$\delta : Q \times \Gamma \times \Gamma \to Q \times \Gamma \times \Gamma \times \{L, R, S\} \times \{L, R, S\}$$

Initial move

$$\delta(q_0, \sqcup, \sqcup) = (q_{acc}, ????)$$

$$\delta(q_0, a, \sqcup) = (q_1, a', a', R, R)$$

$$\delta(q_0, b, \sqcup) = (q_1, b', b', R, R)$$

Conventions:

Regular expressions: $\emptyset$, elements of Σ, ε

ops: $\cup$, $\circ$, *

Some of our minor simplifications over [Sip] simplifications

(jf@cs.ubc.ca, Subject: ... 421 ...)

recognizable = T.m.-recog

DFA's = regular

(1) Talk through!

If L is not regular over $\{a\}$, then

" " " " " $\{a,b\}$

Pf: (a) Myhill-Nerode

(b) Get 3!

L_1, L_2 are regular $\Rightarrow$ $L_1 \cap L_2$ is regular

$\Rightarrow$

If L_1 is regular, but / and $L_1 \cap L_2$ is not regular $\Rightarrow$ L_2 is not regular

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Take $L_2 = L$, $L_1 = \{a\}^*$

$\{a\}^*$ regular, if L regular over $\{a,b\}$

$\Rightarrow$ $\underbrace{\{a\}^* \cap L}_{\text{lives in}}$ regular over $\{a,b\}$

$\{a\}^* \cap L$ regular over $\{a,b\}$

$\Rightarrow$ $\{a\}^* \cap L$ regular over $\{a\}$

crux ??

$\{a\}^* \cap L \subset \{a\}^*$

If L over $\{a, b\}$ is regular,

then

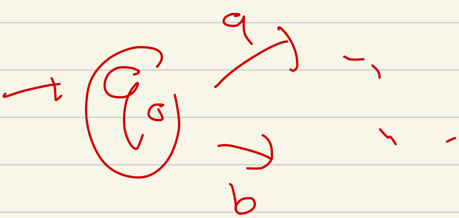
① L described by a regular expression

$$(a \cup b)^* \_\ \_\ \underline{b}.\ \ a\ \_\ \_$$

cross off b's

→ reg exp in a's

② L described by a DFA



cross off b's

T/F

If L_1 & L_2 are recognizable,
then $L_1 \setminus L_2$ is recognizable

Either we prove

or we find a counterexample

$L_1 = \Sigma^*$

L_2 = recognizable (but, say undecidable)

, $ACCEPTANCE_{Python}$, undecidable)

then $L_1 \setminus L_2 = L_2^{comp}$

recognizable = Turing-recognizable

= Python-recognizable

DFA recognizes a language,

but we call such a language

regular

recognizable — to be clarified in

my formula list