CPSC 421/501 Nov 5, 2025

Today: - Decide, recognize, etc.

- Finish PALINDROME on 2-tape

The plan

- Thm: Any L decided by k-tape T.m. can be decided by 1-tape. Note: first t steps of a k-tape algorithm can simulated by $\leq Ct^2$ steps of a 1-tape machine

---

- Universal, delightful, mysterious Tm's

like Python univ NEW NEW

This should seem natural:

if $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$:

(1) accepts $s \in \Sigma^*$ if M reaches q_{acc}

(2) rejects $s \in \Sigma^*$ if M reaches q_{rej}

(3) "loops" $s \in \Sigma^*$ if M never halts,

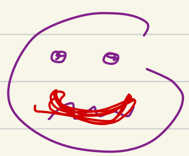
not halting

i.e. never reaches q_{acc} or q_{rej}

M recognizes $\{ s \in \Sigma^* \mid M \text{ accepts } s \}$

M is a <u>decider</u>, L is <u>recognizable</u>, <u>decidable</u>, etc.

$\boxed{ACCEPTANCE_{TM} = ???}$



work...

$\forall$ T.m. M, there is

LangRecBy(M)

$= \{ s \in \Sigma^* \mid$ M accepts s $\}$

$= \{ s \in \Sigma^* \mid$ on input s to M after some finite # of steps, M's computation reaches q_{acc} $\}$

M is a **decider** if $\forall s \in \Sigma^*$, M either rejects s or accepts s, i.e. M always halts

$L \subset \Sigma^*$ is **recognizable** if

$\exists M$ s.t. $L = \text{LangRecBy}(M)$

$L \subset \Sigma^*$ is **decidable** if

$\exists M$ s.t. $\left\{ \begin{array}{l} L = \text{LangRecBy}(M) \text{ and} \\ M \text{ is a decider} \end{array} \right\}$

L is undecidable $\Leftrightarrow$ not decidable

unrecognizable $\Leftrightarrow$ not recognizable

If L is decidable, then

$L^{comp} = \Sigma^* \setminus L$ is decidable

If L is recognizable and

L^{comp}
$= \Sigma^* \setminus L$ " " then

L (and $\Sigma^* \setminus L$) are both decidable

So, we're convinced that

2 T.m. can be "run
in parallel"

(formal proof: given $M = (Q, \Sigma, \Gamma, \delta,$ q_0, q_{acc} q_{rej}

$M' = (Q', \Sigma, \Gamma',$
$\delta', \ldots)$

$\langle \text{blah} \rangle$ = a description of blah

What is

$$\text{ACCEPTANCE}_{TM} = \{ \langle M, i \rangle \mid M \text{ accepts } i \}$$

note: this should be subset of

$\Sigma_{T.M}$ ← some finite alphabet

standardize: $Q = \{1, 2, \dots, |Q|\}$

$$= \{1, 2, \dots, n_Q\}$$

$n_Q = \mathbb{N} = \{1, 2, 3, \dots\}$

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$$

So far

$$Q = \{1, \ldots, n_Q\} = [n_Q]$$

say

$$q_0 \longleftrightarrow 1$$

$$q_{acc} \longleftrightarrow 2$$

$$q_{rej} \longleftrightarrow 3$$

Similarly $\Sigma = \{1, \ldots, n_\Sigma\} = [n_\Sigma]$

$$\Gamma = \Sigma \cup \{\sqcup\} \cup \{\text{other symbols}\}$$

$1, \ldots, n_\Sigma$ $\quad$ $n_\Sigma + 1$ $\quad$ $n_\Sigma + 2, n_\Sigma + 3, \ldots, n_\Gamma$

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$$

Could do this for DFA

Back to Python!

Σ_{ASCII}

$DFA = (Q, \Sigma, \delta, q_0, F)$

$\{1, \ldots, n_Q\}$ $\{1, \ldots, n_\Sigma\}$ $\delta : Q \times \Sigma \to Q$ subset of $[n_Q]$

describe

$\underbrace{n_Q}_{\text{base } 10}, \underbrace{n_\Sigma}_{\text{base } 10}, (\delta) \ldots$

PALINDROME is decidable, specifically on input s, within # steps $O(|s|)$, i.e. $\leq C|s|$ for $|s|$ large on a 2-tape machine: