CPSC 421 Nov 3, 2025

This week: Multitape machines

and $PALINDROME_{\{a,b\}}$

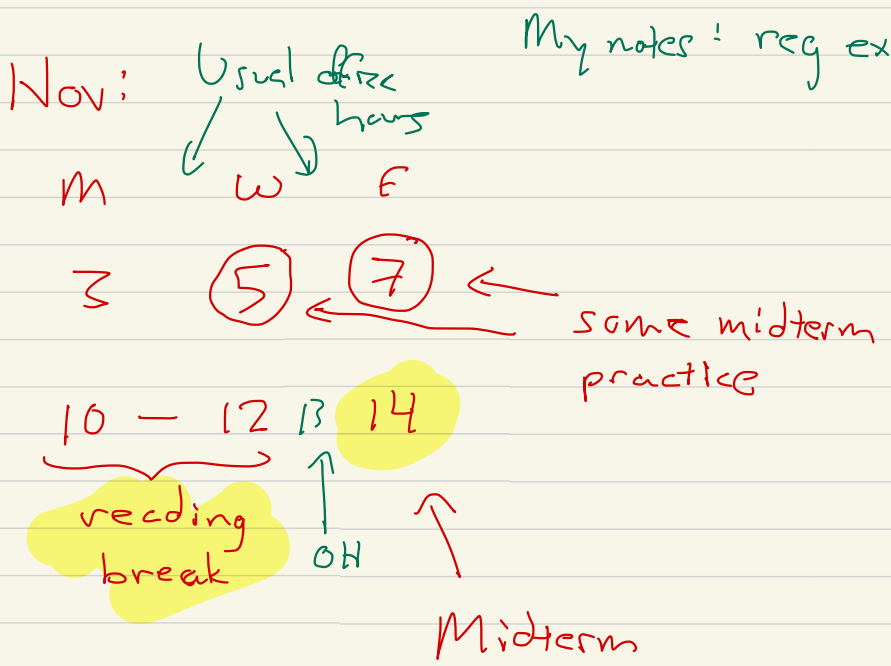
$ADD = \{ x\#y\#z \mid x+y=z \}$

$MULT = \{ x\#y\#z \mid xy=z \}$

and universal T.M.'s

A TM decides, recognizes, accepts, halts, ...

Delightful & mysterious Tm's



SWNG 222, 45 min

Remain Outside the classroom until called in.

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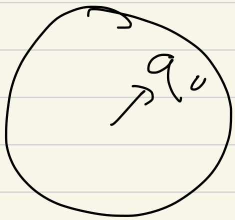
1 two-sided 8.5" × 11" note sheet

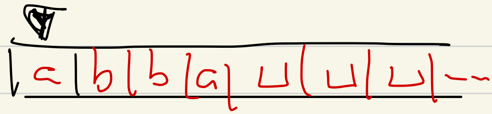
Recall 1-tape $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

Multitape: k-tape

$$\delta : Q \times \Gamma^k \longrightarrow Q \times \Gamma^k \times \{L, R, S\}$$



tape 1

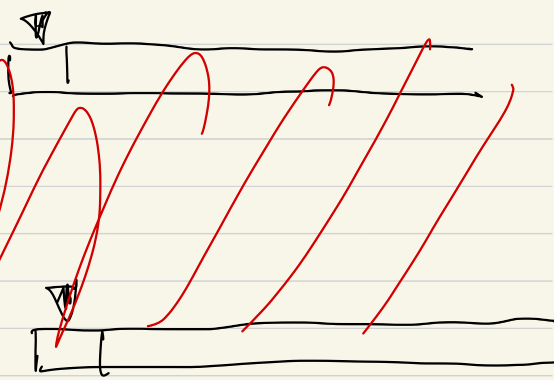


← input

tape 2

⋮

tape k



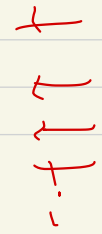
$$\begin{array}{r} {}^{1}1\,{}^{1}2\,3\,4 \\ +\ 5\,6\,7\,8 \\ \hline 6\,9\,1\,2 \end{array}$$

$\Sigma = \{0, \ldots, 9, \#\}$

$$\text{ADDITION} = \{ x \# y \# z \mid x + y = z \}$$

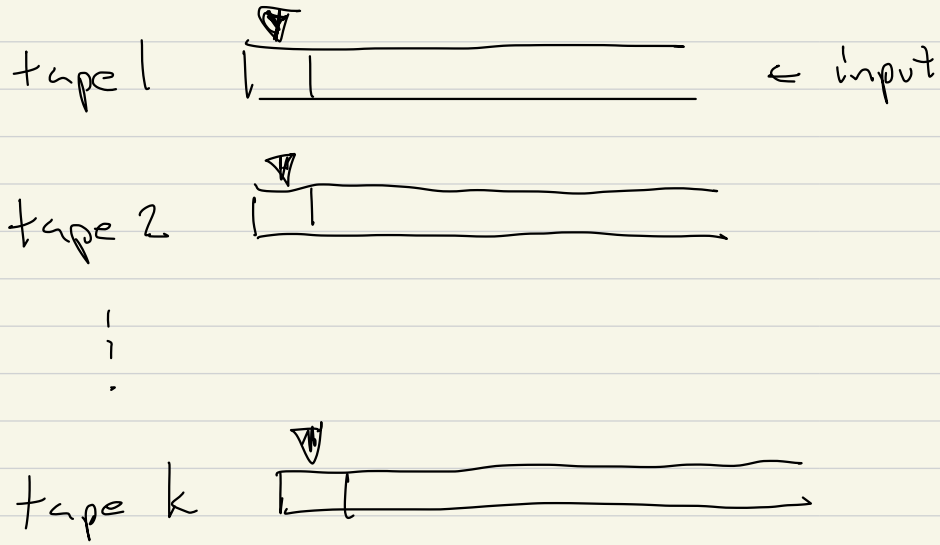
a decision problem version of addition

$$\begin{array}{r} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ \times\ 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ \hline \end{array}$$



A language $L \subset \Sigma^*$ is recognized by a k-tape Turing machine (k fixed, $k = 1, 2, \ldots$) if

$$\delta : Q \times \Gamma^k \longrightarrow Q \times \Gamma^k \times \{L, R, S\}^k$$



new

S = stay

L = left

R = right

PALINDROME

$\{a, b\}$

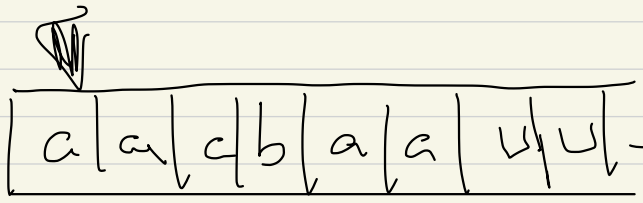
(there is a linear time algorithm)

time = # of steps need

linear = linear function of $|i|$

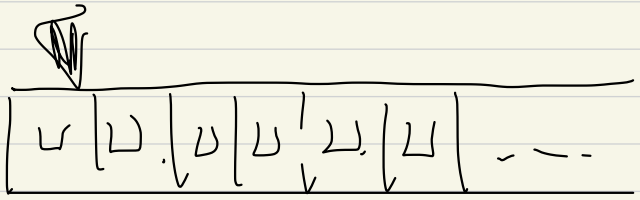
i = input

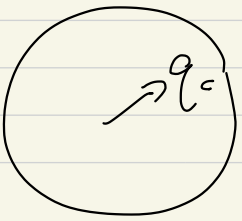
$|i|$ = length of input



initial state

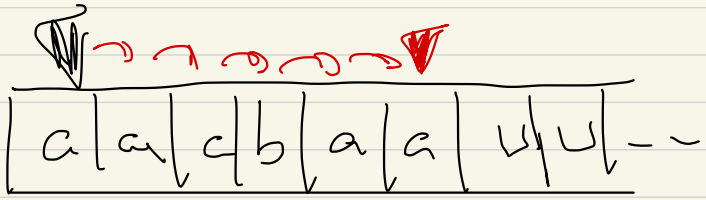
(step 1)





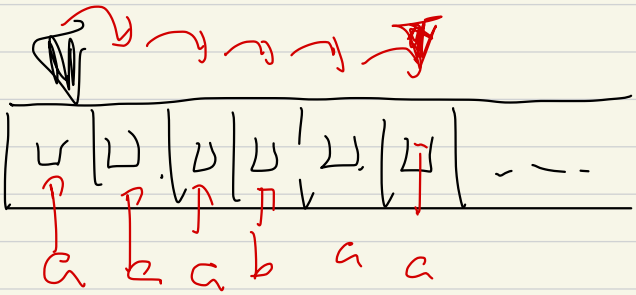
Alg:

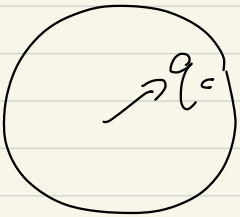
Phase 1:



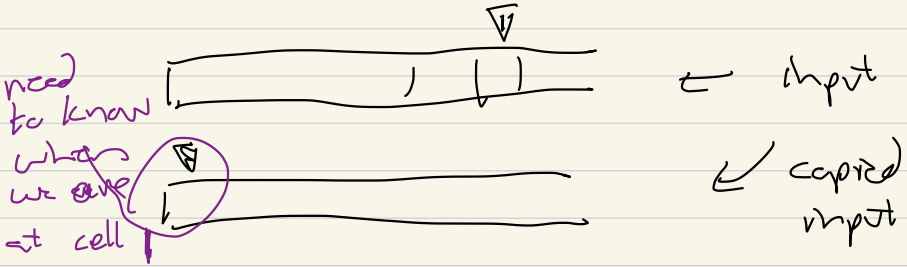
initial state

(step 1)

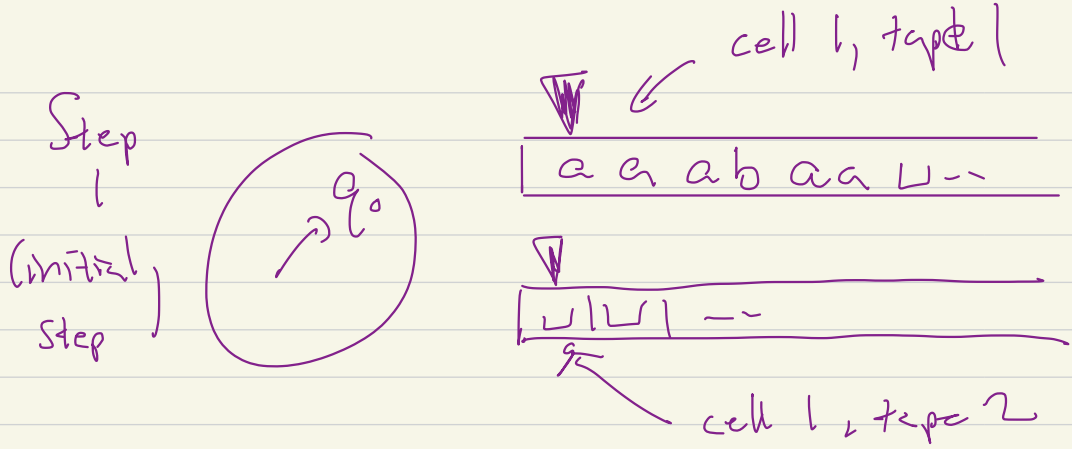




Phase 2



Phase 3 compare



$\Sigma = \{a, b\}$, $\Gamma = \{a, b, \sqcup, a', b'\}$

$$\delta : Q \times \Gamma^2 \longrightarrow Q \times \Gamma^2 \times \{L, R, S\}^2$$

tape

Cell $\boxed{\sqcup|}$ Input $\Longleftrightarrow$ input $\in \Sigma$

Since $\varepsilon \in$ PALINDROME $\{a, b\}$

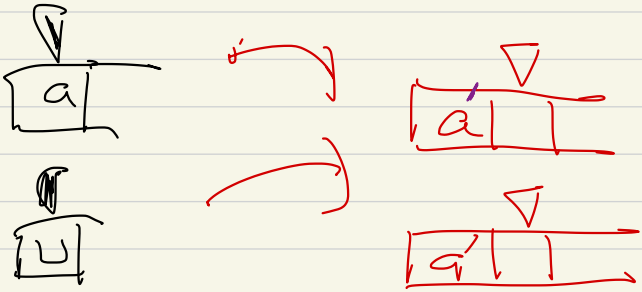
Running δ left

$\delta(q_0, (\sqcup, \sqcup))$

$\delta(q_0, \sqcup, \sqcup)$ = $(q_{acc}, ?, ?, ?)$

q_{acc}

for CPSC 421, Fig



$\delta(q_0, a, \sqcup) = (q_1, a', a', R, R)$

where q_1 means the right moving part of Phase 1

q_1 — δ

$$\delta(q_0, b, \sqcup) = (q_1, b', b', R, R)$$

So far...

(1) High-level description 

(2) Implement, but q_1 = describe:

q_1 means "the right moving part of Phase 1"

(3) Write down δ function values