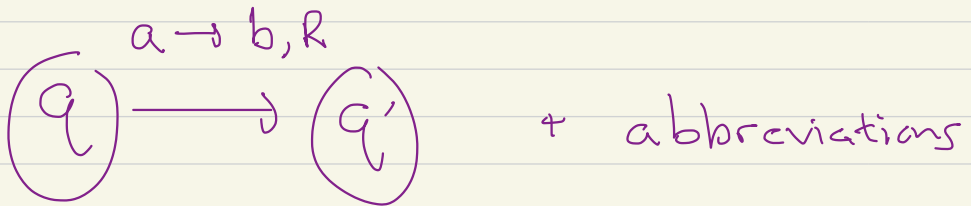


CPSC 421/501

Oct 31, 2025

We are building a decider for

PALINDROME $\{a,b\}^*$



Rem: recognize, decide, etc.

comes from q_{acc}, q_{rej}

Sipser likes

$a a \underbrace{q}_{\underbrace{q}} b b c$

efficient to see what
a T.M. is doing

≈ 1 week

Turing Machines :

$$M = (\underline{Q}, \underline{\Sigma}, \underline{\Gamma}, \delta, \underline{q_0}, \underline{q_{acc}}, \underline{q_{rej}})$$

1-tape $\delta : Q \times \Gamma \rightarrow \underbrace{Q \times \Gamma \times \{L, R\}}$

non-deterministic

$$\delta : Q \times \Gamma \rightarrow \text{Power}(Q \times \Gamma \times \{L, R\})$$

2-tape, k-tape $k \text{ is finite}$

$$\delta : Q^{??} \times \Gamma^{??} \rightarrow Q^{??} \times \Gamma^{??} \times \{L, R, S\}^{??}$$

$L = \text{left}, R = \text{right}, S = \text{stay}$

Back to Hurdert: Delightful, mysterious

PALINDROME $\{a, b\}$

$$= \left\{ s \in \{a, b\}^* \mid s^{\text{rev}} = s \right\}$$

Algorithm

Step 1: Compare 1st and Last symbol
of input $S = \sigma_1 \dots \sigma_n$

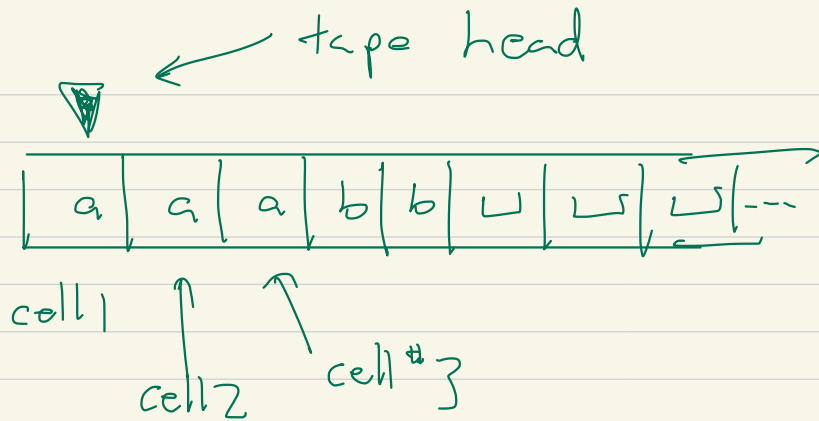
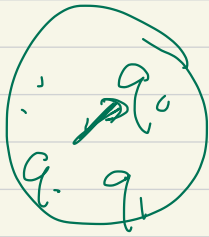
Step 2: Compare 2nd sym & 2nd to
last symbol

$$\Sigma = \{a, b\}$$

In parallel:

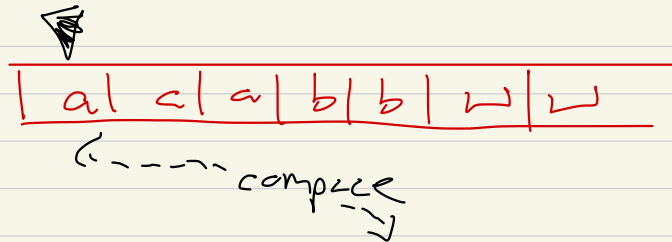
$$\Gamma = \{a, b, \sqcup, \underbrace{a', b', \bar{a}, \bar{b}}_{\text{any finite list}}\}$$

-4-

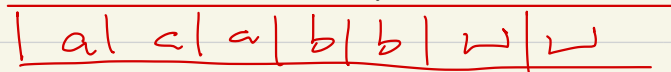


input is $a a a b b \dots$

Phase 1



could we
write



Phase 1.1: move right, but mark
the first cell

Option 1: first $a \rightarrow \bar{a}$
first $b \rightarrow \bar{b}$
first $L \rightarrow qacc$

Alternative:

write L in cell 1



L	a	a	b	b	␣	␣
---	---	---	---	---	---	---

$a, b \rightarrow R$ shorthand

$a \rightarrow a, R$

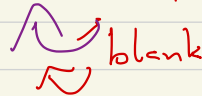
$b \rightarrow b, R$

$a \rightarrow \perp$

$b \rightarrow R$



L	a	a	b	b	␣	␣
---	---	---	---	---	---	---



$a, b \rightarrow R$

$U \rightarrow U, L$

Q moving to right R

Q saw b, just read L and moved left

$b \rightarrow U, R$
 $a \rightarrow U, R$

$a, b \rightarrow R$

Q moving to right R
saw a

$U \rightarrow U, L$

$b \rightarrow U, L$
 $a \rightarrow$ doesn't matter

Q_c

Q saw a just read L and moved left

$U \rightarrow$ doesn't matter

$U \rightarrow ??$

$U \rightarrow ??$

$b \rightarrow$

Q move to left

Q_{acc}

Q_{rej}