CPSC 421/501 Oct 20, 2025

Today: $L = \{ a^1, a^4, a^9, \ldots \}$

$\mathrm{AccFut}_L(a^n)$

How to choose $n_1, n_2, \ldots$

to most easily see that

L is not regular

What about $L = \{ a^n b^n \mid n \in \mathbb{N} \}$

What about min # states to

recognize $L = ab\Sigma^* ab\Sigma^*$

Later today or Wednesday:

Turing machine:

$$(Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$$

$\rightsquigarrow$ \$$10^6$ USD for P vs. NP ...

HW 7

Prob (4)

over an alphabet, Σ, design a $O(q^2|\Sigma|)$

Paris in the the spring

what's wrong

Auden

L is (non-regular, we know...)

$$L = \{ a^1, a^4, a^9, a^{16}, \dots \}$$

Claim: For all $m, m' \in \mathbb{Z}_{\geq 0}$, $m \neq m'$,

$$\text{AccFut}_L(a^m) \neq \text{AccFut}_L(a^{m'})$$

What is an easier way to prove that

L is non-regular

than examining $\text{AccFut}_L(a^m)$ $\forall m$?

J.Y. !

look at $m = 1, 4, 9, 16, \dots$

$$\mathrm{AccFut}_L(a) = \{\varepsilon, a^3, a^8, a^{15}, \dots\}$$

$$\mathrm{AccFut}_L(a^4) = \{\varepsilon, a^5, \dots\}$$

$$\mathrm{AccFut}_L(a^9) = \{\varepsilon, a^7, \dots\}$$

2nd to shortest word is

$3, 5, 7, 9, \dots$ the gaps

$\Sigma = \{a\}$

Calculation scratch sheet

$\mathrm{AccFut}_L(a)$

$= \{ s' \in \Sigma^* \mid as' = a^1, a^4, a^9, \dots \}$

$= \{ s = a^k \mid aa^k = a^1, a^4, \dots \}$

$\underbrace{aa^k}_{a^{k+1}} = a^{n^2}$ $\quad n \in \mathbb{N}$

$a^{k+1} = a^{n^2}$

$k+1 = n^2$

$k = n^2 - 1, \quad n = 1, 2, 3, \dots$

$k = 1^2 - 1, 2^2 - 1, 3^2 - 1$

$= 0, 3, 8, 15, \ldots$

$\Rightarrow \quad s' = a^0, a^3, a^8, a^{15}, \ldots$

On midterm / final you'd write

$$\text{AccFut}_L(a) = \{\varepsilon, a^3, a^8, a^{15}, \ldots\}$$

$$\text{AccFut}_L(a^4) = \{\varepsilon, a^5, \ldots\}$$

$$\text{AccFut}_L(a^9) = \{\varepsilon, a^7, \ldots\}$$

$\vdots$

similarly,

(thanks J.S.)

$$\text{AccFut}_L(a^{k^2}) = \{\varepsilon, a^{(k+1)^2 - k^2}, \ldots\}$$

$$= \{\varepsilon, a^{2k+1}, \ldots\}$$

so 2^{nd} shortest word

in

$$\mathrm{AccFut}_L(a^{k^2}) \text{ is }$$

$$a^{2k+1}$$

hence for $k = 1, 2, 3, \ldots$

all sets

$$\mathrm{AccFut}_L(a^{k^2}) \text{ are distinct,}$$

hence L is non-regular

N.B. !   gaps

1, 4, 9, 16, 25

prove (the easy way ..)

$L = \{ a^n b^n \mid n \in \mathbb{N} \}$ — is this regular?

Now: we want $s_1, s_2, \ldots$ and show

$\left.\begin{array}{l} \mathrm{AccFut}_L(s_1), \\ \mathrm{AccFut}_L(s_2), \\ \vdots \end{array}\right\}$ are all distinct

---

We can take

1. $s_1 = a, \quad s_2 = aa, \quad s_3 = a^3, \ldots$

$s_n = a^n$

Ju:

(2) $S_1, S_2, \ldots,$

$S_1 = ab$

$S_2 = a^2b$

$\vdots$

$S_n = a^nb$

HW.. of all 2^n strings in Σ^*

$\{a, b\}^n$

$\ldots$

Consider:

$L = ab\Sigma^*ab\Sigma^*$

Careful here ...

some may write

$\mathrm{Accfut}_L(\varepsilon) = L$ itself

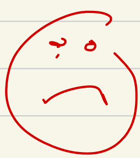
$= ab\Sigma^*ab\Sigma^*$

$\mathrm{Accfut}(abb) = \Sigma^*ab\Sigma^*$

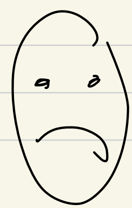
$AcFut_L(a) = \{ bab, \ldots$

$\ldots \quad \ldots$

$AcFut_L(abba) = \{ \ldots \}$

~~Obviously~~, these are all distinct  

OR

~~Obviously~~ $ab\Sigma^*ab\Sigma^* \neq \Sigma^*ab\Sigma^*$ 

Make sure on the midterm/final you fully justify why if you think that two sets

$$\left.\begin{array}{l} \mathrm{Accept}_L(S_1) \\ \mathrm{Accept}_L(S_2) \\ \vdots \end{array}\right\} \text{ are distinct,}$$

then you have a short justification

Class Ends

Prove that

1) $L = \{a, a^4, a^9, a^{16}, \ldots\}$

is non-regular over $\Sigma = \{a, b\}$

Say that we know that

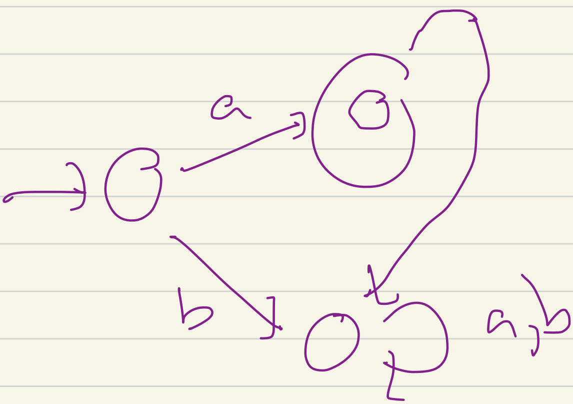
2) L is non-regular over $\Sigma = \{a\}$

Proof: Say that, for the sake of contradiction L is regular over $\Sigma = \{a, b\}$

$(S, A,)$ ---

use $M = N$.

$L = \{a\}$



DFA