CPSC 421/501 Oct 17, 2025

Finish Myhill-Nerode:

$\mathrm{AccFut}_L(s)$ $s \in \Sigma^*$

$$\overset{\text{def}}{=} \{ s' \mid ss' \in L \} \subset \Sigma^*$$

$\{ s' \mid ss' \in L \}$ is a language over $\Sigma$

Claim:

— If $L$ is regular, then:

1. $$q_{min} = \left| \overset{\#}{\{} \mathrm{AccFut}_L(s) \mid s \in \Sigma^* \} \right|$$

is the minimum # of

$q_{min} = q_{min}(L)$ [is finite]

in any DFA recognizing $L$

2 This DFA is unique

and can be "built"

from "knowledge" of

$\{ \text{AccFut}_L(s) \mid s \in \Sigma^* \}$

2a with no needless states

— If $L$ is non-regular, then

"$q_{min}(L) = \infty$"

Admin:

Some bonus problems will not have solutions provided.

All such problems WILL NOT appear on the 2025 midterm and final.

Apologies for the long
Homework 7 ...

- Some teach by "teaching"
- Some teach by "pointing the way"
- I (sometimes) teach by
giving so much busy work
that you have no choice but
to (1) learn, and
(2) ask for shorter
homework assignments

Myhill-Nerode! Last time

Examples?

Applications 1:

Min # states in

$\Sigma^*$, $abba$, $abba^*$,

$a^2 \cup b$ or $aa \cup b$ or $(a \circ a) \cup b$

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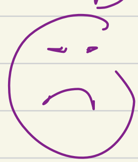
$as' = aa$, $as' = b$

$\cancel{a}s' = \cancel{a}a$ $\cancel{a}s' = b$

$s' = a$



What about any regular language?

How time complex, ... ?

1

$$q_{min} = \left| \overset{\#}{\{} \mathrm{AccFut}_L(s) \mid s \in \Sigma^* \} \right|$$

$q_{min} = q_{min}(L)$ is the minimum # of states [if $L$ is regular]

and

**MOREOVER**

$q_{min} = q_{min}(L)$ is finite

$\Longleftrightarrow$ $L$ is regular

thanks A.K. and M.Y. (in 2025)

Example: Non-Regular Language

$$L = \{ a^{n^2} \mid n \in \mathbb{N} \}$$

$$= \{ a^1, a^4, a^9, a^{16}, \ldots \}$$

$$L = \{ a^n b^n \mid n \in \mathbb{N} \}$$

[Sipser] $\{ 0^n 1^n \mid n \in \mathbb{N}$ or $\mathbb{Z}_{\geq 0} \}$

$$\{ a^n b^m \mid m \geq n, \ m, n \in \mathbb{N} \}$$

Example: $\Sigma = \{a\}$

$$L = \{ a^{n^2} \mid n \in \mathbb{N} \}$$

$$L = \{ a^1, a^4, a^9, a^{16}, \dots \}$$

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$\mathrm{AccFut}_L(s)$ $\quad s \in \Sigma^*$

$$\mathrm{AccFut}_L(\varepsilon) = \{ s' \mid \underbrace{\varepsilon s'}_{s' \in L} \in L \} = L$$

Rem: $\mathrm{Accfut}_L(\varepsilon) = L$ always

$\mathrm{Accfut}_L(a)$

$= \{ s' \mid as' \in L \}$

$= \{ a^k \mid a a^k \in L \}$

$a^{k+1} \in L$

$a^{k+1} \in \{ a, a^4, a^9, a^{16}, \ldots \}$

$k = 0, 3, 8, \ldots$

$$\mathrm{Accfut}_L(\varepsilon) = \{a^1, a^4, a^9, a^{16}, \ldots\}$$

$$\mathrm{Accfut}_L(a) = \{a^0, a^3, a^8, a^{15}, \ldots\}$$

$$\mathrm{Accfut}_L(a^2) = \{a^2, a^7, a^{14}, \ldots\}$$

$$\mathrm{Accfut}_L(a^3) = \{a^1, \ldots\}$$

$$\underbrace{a^1, a^4}_{3}, \underbrace{\phantom{a}a^9}_{5}, \underbrace{\phantom{a}a^{16}}_{7}$$

Larger gaps $\to \infty$ $\Rightarrow$ non-reg

$$\mathrm{AccFut}_L(a^{1+1}) = \{a^{4-1-1}, \ldots$$

$a^2$

$$\mathrm{AccFut}_L(\quad) =$$

$$\mathrm{AccFut}_L(\quad) =$$

To be completed (correctly) on Monday

# CLASS ENDS

m.y. sugg

$\text{Accept}_L \quad a \quad = \{\varepsilon, C$

$\qquad a^4 \quad = \{\varepsilon, C$

$\qquad a^9$

$\qquad \vdots$

$a^{n^2-1} \qquad n \in \mathbb{N}$

J.Y.