CPSC 421/501 Oct 10, 2025

- NFAs → DFAs (Oct 8)

- Regular Expressions:

  [Sip]: $\emptyset$, ε, elements of Σ

  plus $\cup$, $\circ$, *

  [Could add: $^+$, $\neg$, $\wedge$, ...]

- Regular expressions describe regular languages (with proof)

- Vice versa: a cautionary tale...

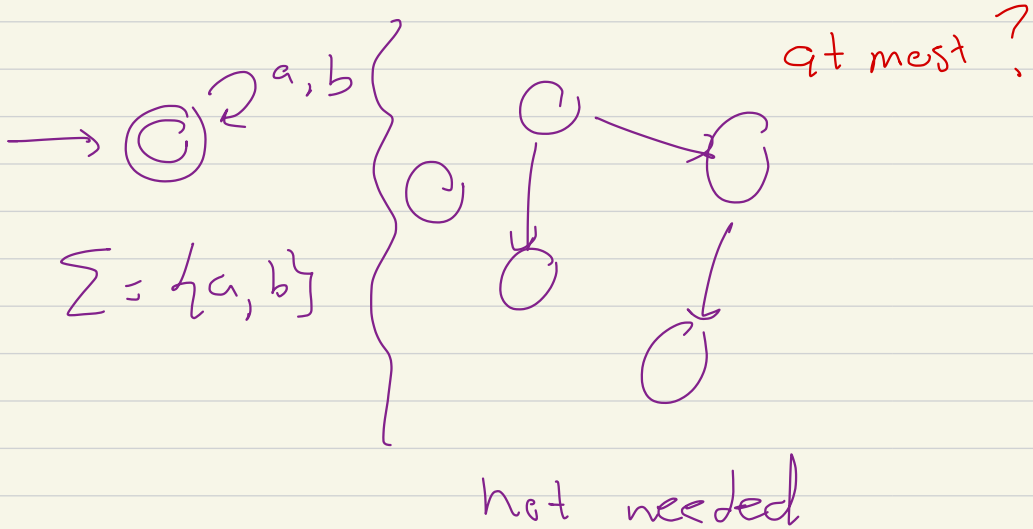
→ in §1.3 [Sip]

Say you have an NFA with

$M = (Q, \Sigma, \delta, q_0, F)$ and

$|Q| = k$.

How large — how many states could an equivalent DFA have?



$\Sigma = \{a, b\}$

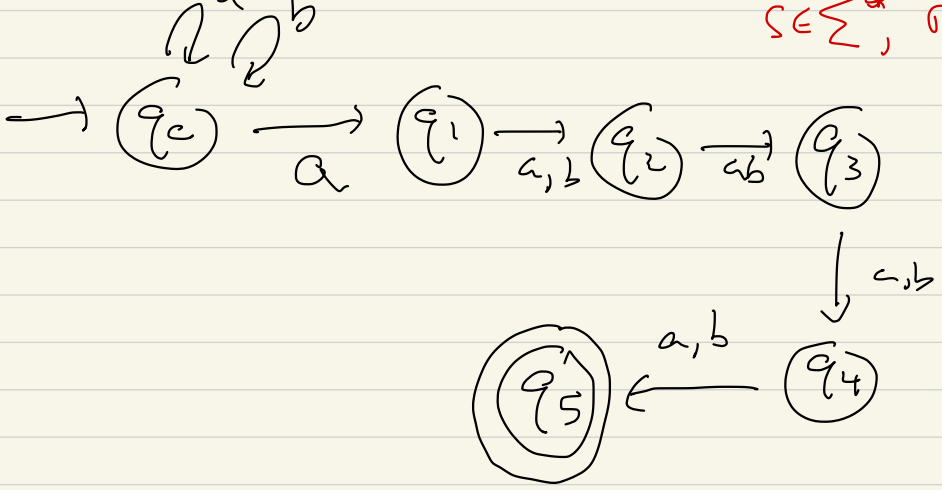
$(\Sigma^*) \circ (\{a\}) \circ \Sigma^4$

$\Sigma^* a \Sigma^4$ ← regular exp.

$L = \{ s \mid$ 4th to last letter/symbol is an $a \}$

NFA

$= \{ s a \sigma_1 \sigma_2 \sigma_3 \sigma_4 \mid s \in \Sigma^*, \sigma_1, \ldots, \sigma_4 \in \Sigma \}$

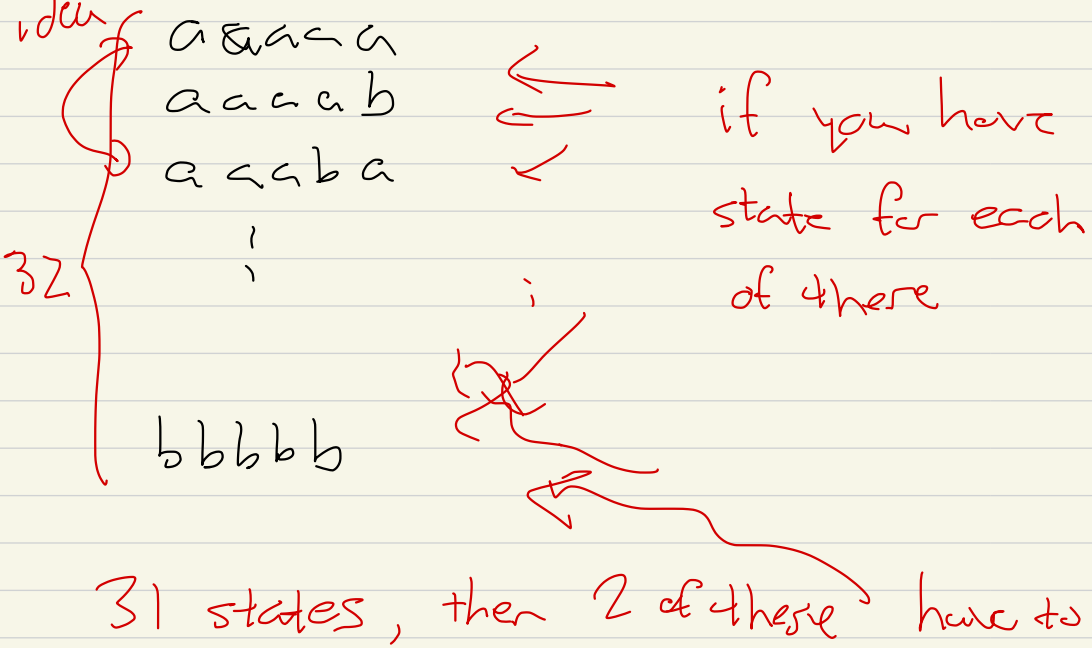


blah
blah
blah

$a \sigma_1 \sigma_2 \sigma_3 \sigma_4$

How large is a min size DFA

for $L = \Sigma^* a \Sigma^4 = \Sigma^* a \Sigma \Sigma \Sigma \Sigma$



31 states, then 2 of these have to wind up in the same state

NFA → 6 states

DFA → 2^5 states

For any $L \subseteq \Sigma^*$

$$L^{rev} = \{ s^{rev} \mid s \in L \}$$

and

s^{rev} ! means

$$(\sigma_1 \cdots \sigma_n)^{rev} = \sigma_n \sigma_{n-1} \cdots \sigma_1$$

$$(aab)^{rev} = baa$$

$$(abba)^{rev} = abba \text{ (palindrom)}$$

$$(\Sigma^* a \Sigma^4)^{rev} = (\Sigma^4)^{rev} (a)^{rev} (\Sigma^*)^{rev}$$

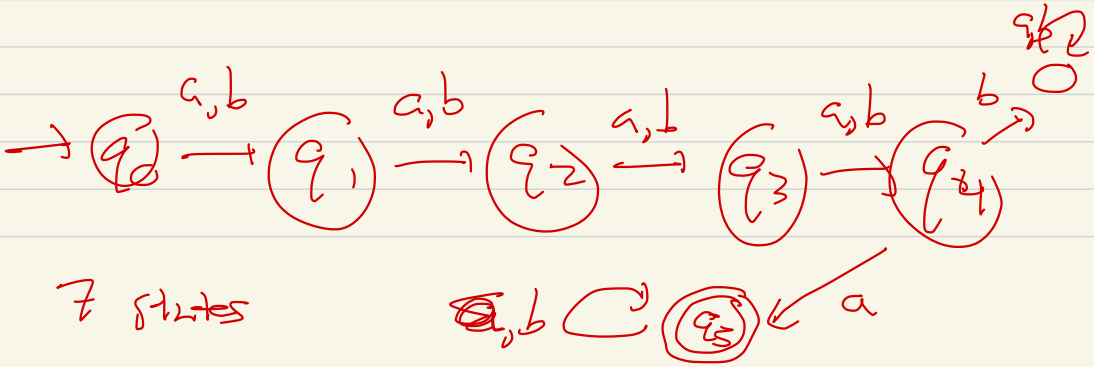
$$(L_1 L_2 L_3)^{rev} = L_3^{rev} L_2^{rev} L_1^{rev}$$

$$(L_1 \circ L_2 \circ L_3)^{rev} = L_3^{rev} \circ L_2^{rev} \circ L_1^{rev}$$

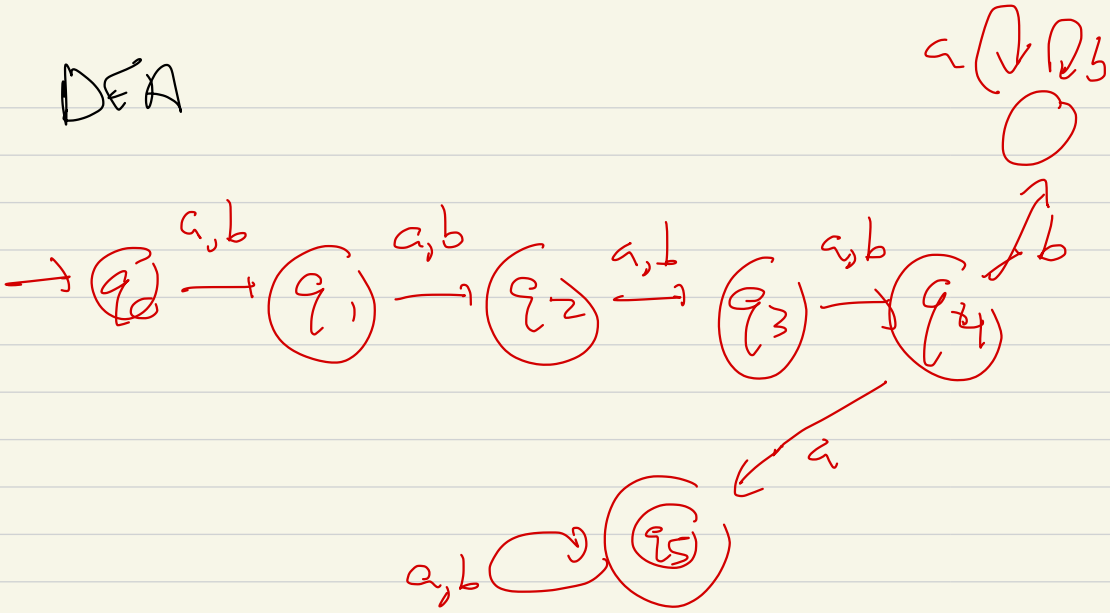
Example

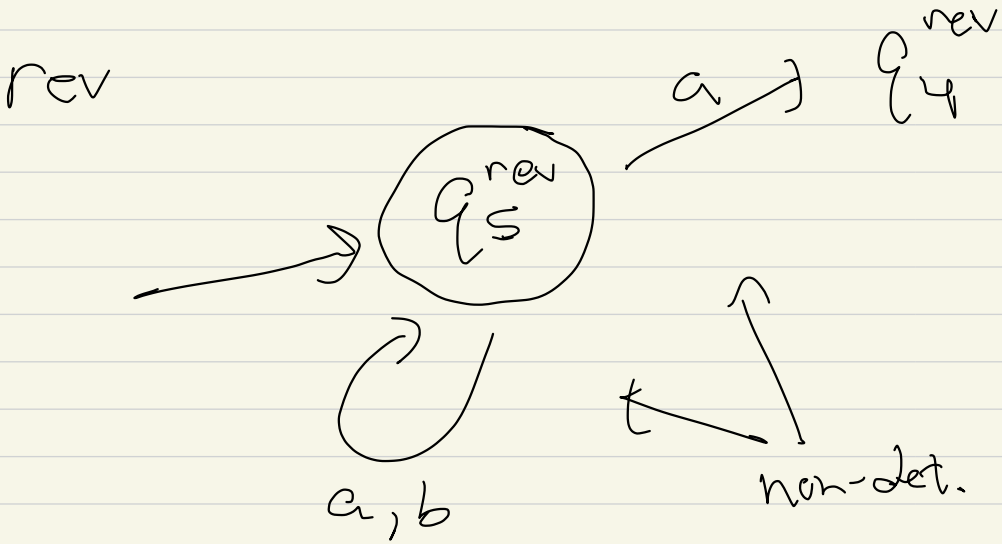
$$L = \{ s \mid 5^{th} \text{ to last letter/symbol is an "a"} \}$$

$$L^{rev} = \{ s \mid 5^{th} (\text{from first}) \text{ letter/symbol is an "a"} \}$$



DFA





(NFA)

Regular Expression:

$\Sigma \leftarrow$ alphabet

Rules:

Primitives: (1) $\emptyset$, (2) $\{\sigma\}$ or σ for any $\sigma \in \Sigma$

(3) $\{\varepsilon\}$ or ε

Building Rules: if R_1, R_2 regular expressions, then so are

(4) $(R_1) \cup (R_2)$ (5) $(R_1) \circ (R_2)$ (6) $(R_1)^*$

$\Sigma = \{a, b\}$

e.g.

$$(\{a\} \cup \{b\})^* = \{a\}^*$$

$$(\{a\} \cup \{b\}) \circ (\{a\} \cup \{b\}) \circ$$

$$\circ (\{a\} \cup \{b\}) \circ (\{a\} \cup \{b\})$$

$\{a\} \cup \{b\} \xleftarrow{\text{write}} \Sigma$, $L \circ L \circ L \circ L = L^4$ (write)

$$\Sigma^* \circ \{a\} \circ \Sigma \circ \Sigma \circ \Sigma \circ \Sigma$$

$$\Sigma^* \circ \{a\} \circ \Sigma^4 = \Sigma^* \circ (a) \circ \Sigma^4$$

$$= \Sigma^* a \Sigma^4$$