CPSC 421/501 Oct. 8, 2025

- NFAs $\longrightarrow$ DFAs

- Regular Expressions:

  [Sip]: $\emptyset$, ε, elements of Σ

  plus $\cup$, $\circ$, *

[Could add: $+$, $\neg$, $\cap$, ...]

- Regular expressions describe regular languages (with proof)

- Vice versa: a cautionary tale...

Idea: L_1 regular DFA M_1

L_2 " " M_2

$L_1 \circ L_2$ is regular

View M_1 as NFA

" M_2 " "

Trick (uses <u>N</u>FA)

- build DFA for $L_1 \circ L_2$
- NFA implementation of $M_1 \circ M_2$

Caution! # states in "$M_1 \circ M_2$"

= (# states in M_1) + (# states in M_2)

NFA → DFA

states in resulting DFA 

Trick:

$$NFA = (Q, \ldots)$$

$$\Downarrow$$

$$DFA = (Power(Q), \ldots)$$

Idea!

State set of DFA is Power(Q), intuition is

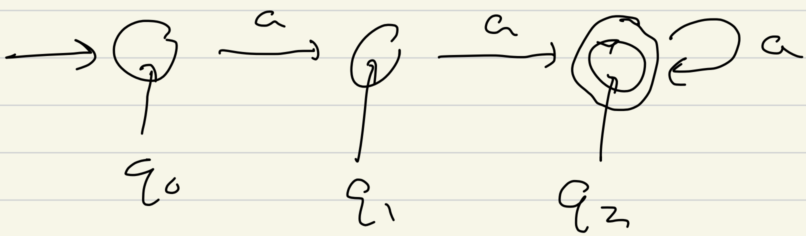
$$t \in Power(Q), \quad t \subset Q$$

In state t of the DFA means in the NFA we could get to all elements of t

Rem: $|\text{Power}(Q)| = 2^{|Q|}$

$=$

Say NFA:



$\leftarrow$ NFA

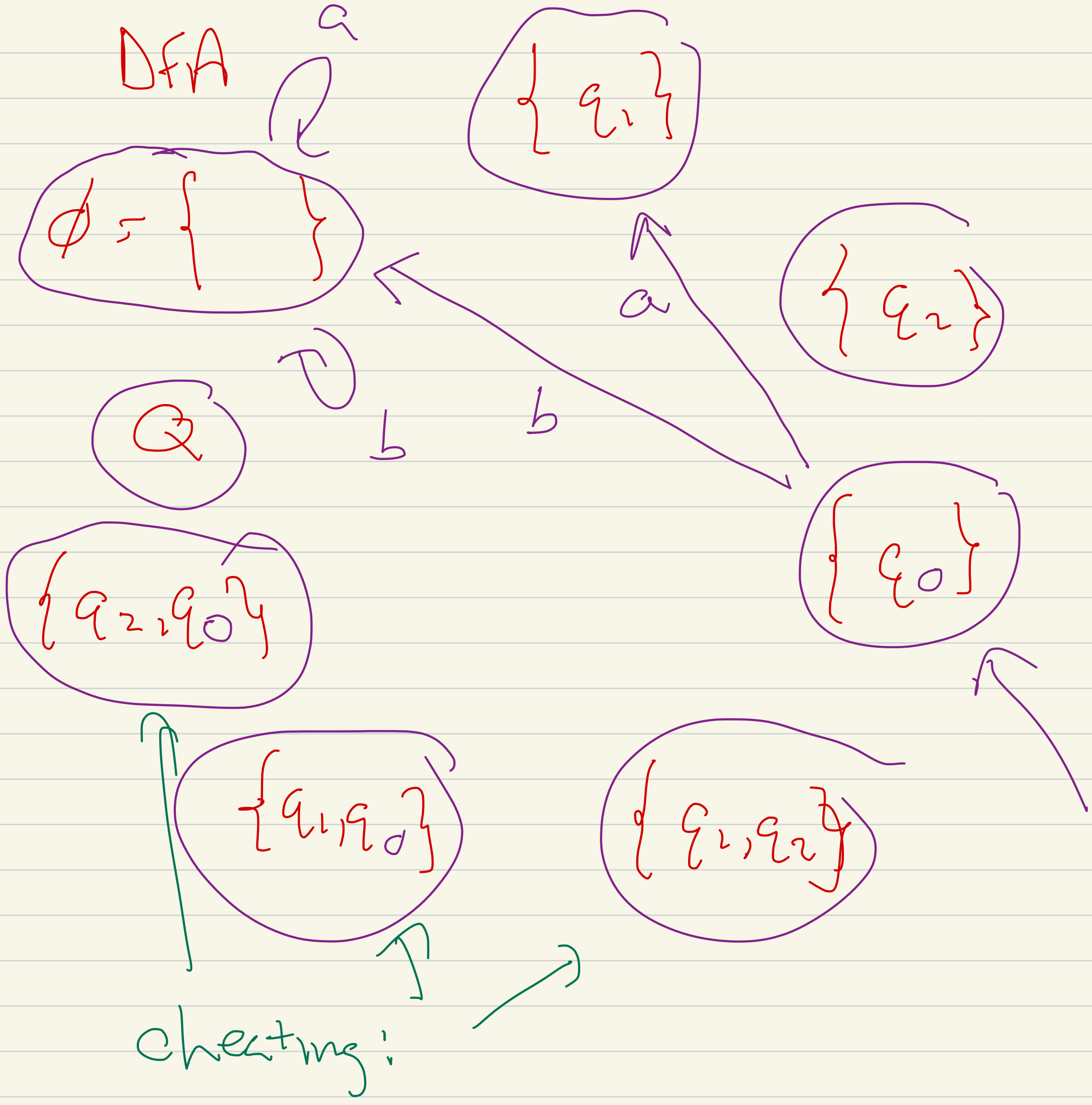
$\Sigma = \{a, b\}$

In NFA

aabbaa

q_0

Initial state before reading anything, is $\{q_0\}$



Initial state: $\{q_0\}$

$\delta(\{q_0\}, b) =$ set of all states we can reach: $\emptyset$

$\delta(\emptyset, \sigma) = \emptyset$

any $\sigma \in \Sigma$

if we can't be in any state

input ab

$$\{q_0\} \xrightarrow{a} \{q_1\} \xrightarrow{b} \emptyset$$

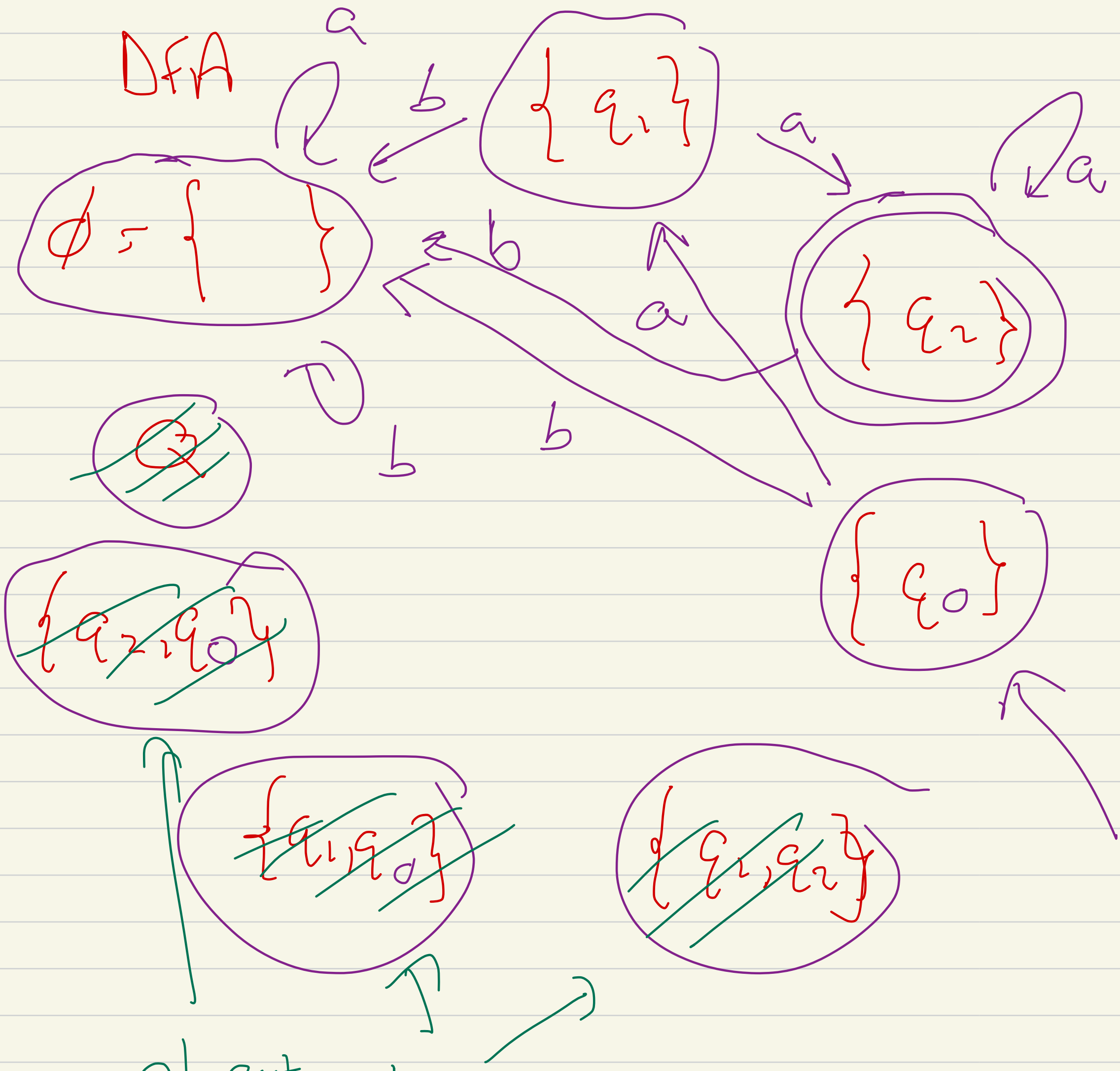
a

no b arrow

$\emptyset \in$ Power$(\{q_0, q_1, q_2\})$

$\emptyset \subset \{q_0, q_1, q_2\}$

$\{\ \} \subset \{\ \}$



cheating:

we don't really use

2-element subsets
3-element subset

Warning:

$\{q_1, q_2\}$ would be accepting, if we needed it

$t \subset Q$, $t \in Power(Q)$

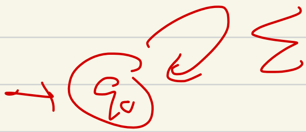
t is accepting iff t contains an accepting state of the NFA

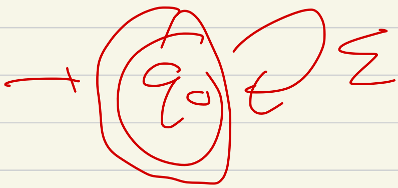
$NFA = (Q, \Sigma, \delta, q_0, F)$

$$\delta : Q \times \underbrace{\Sigma_\varepsilon}_{\Sigma \cup \{\varepsilon\}} \longrightarrow Q$$

Let

$$L = \begin{cases} \emptyset & \text{if } P = NP \\ \Sigma^* & \text{if } P \neq NP \end{cases}$$

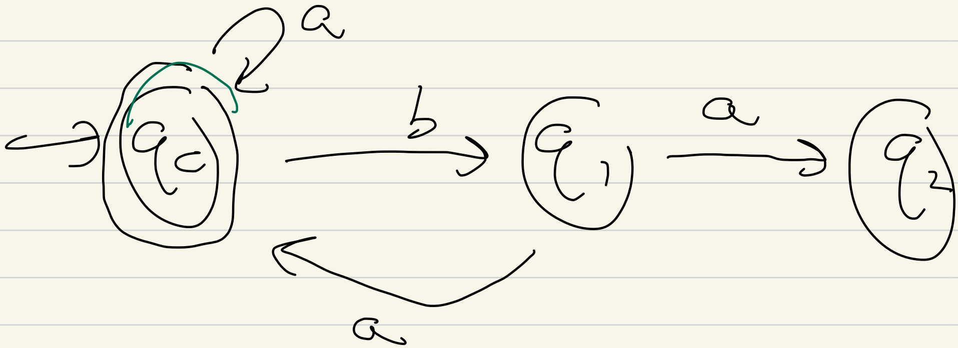
$\emptyset$: 

Σ^* : 

then L has a 1-state DFA that recognizes it.

How to build it ... ???

NFA:

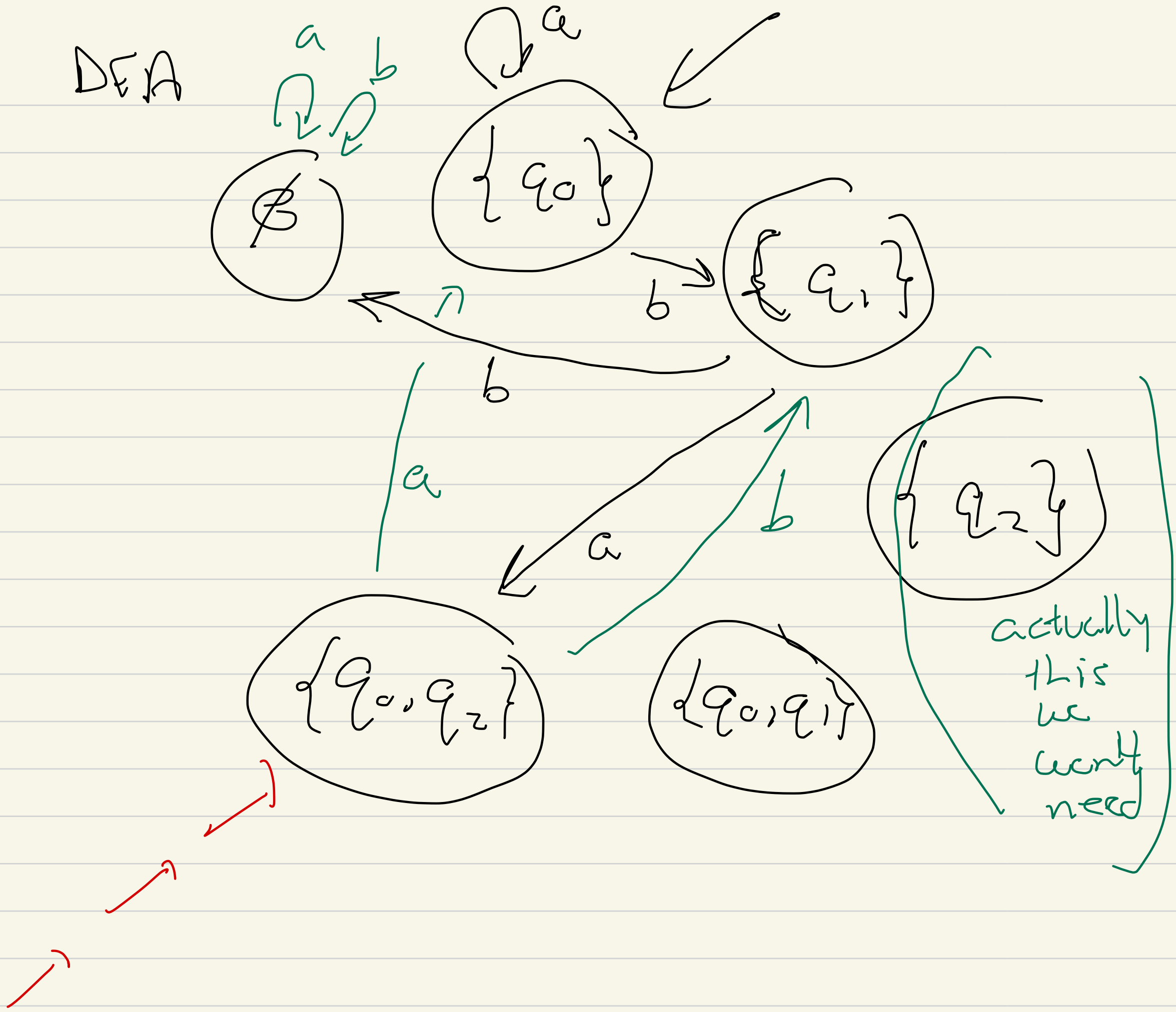


$$(Q, \Sigma, \delta, q_0, F)$$

Sample input aaba....

$$\{q_0\} \xrightarrow{a} \{q_0\} \xrightarrow{a} \{q_0\} \xrightarrow{b} \{q_1\} \xrightarrow{a} \{q_0, q_2\}$$

DFA



$$\left(\text{Pow}(Q), \Sigma, \tilde{\delta}, \{q_0\}, \tilde{F} \right)$$

$$\tilde{F} = \left(\{q_0\}, \{q_0, q_2\} \right)$$