CPSC 421/501 Oct 6, 2025

Last time:

① L_1, L_2 regular $\Rightarrow$ $L_1 \cap L_2, L_1 \setminus L_2, L_1 \cup L_2, \ldots$ are regular. What about $L_1 \circ L_2$? L_1^*?

② $\{a^p \mid p \text{ prime}\} \cup \{b^p \mid p \text{ prime}\}$ is non-regular, since its intersection with $\{a\}^*$ is $\{a^p \mid p \text{ prime}\}$

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③ Today: L is regular $\Rightarrow$ L^* is regular.

Example: $\{a^5, a^7\}^*$

Why: we strengthen DFAs to get NFAs

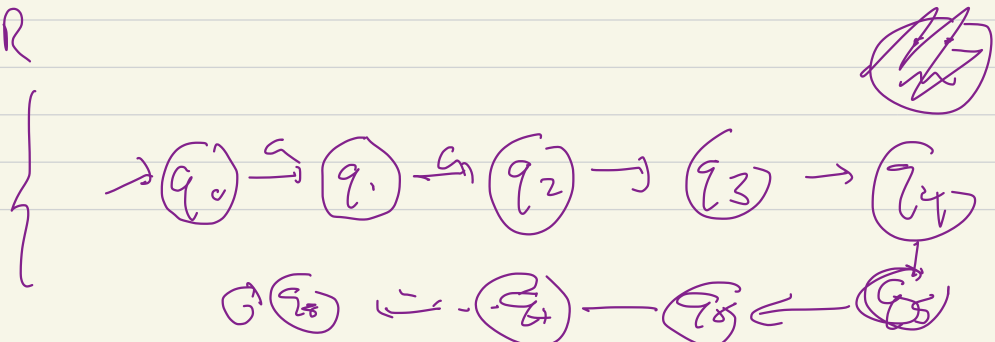
$\{a^5, a^7\}$:

q_i = state that you reach on a^i (except $i \geq 8$)

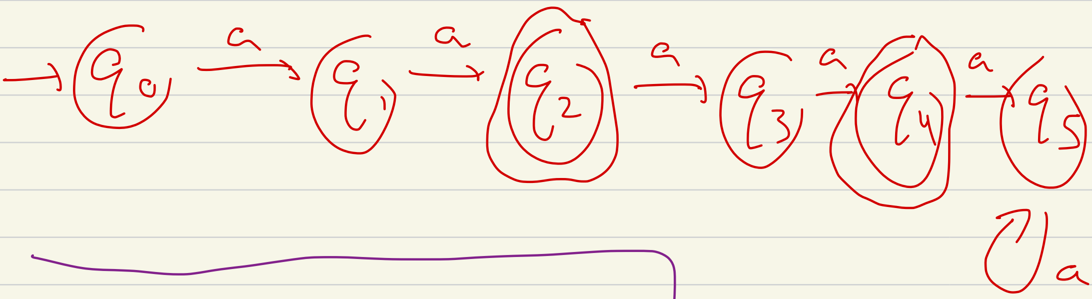
$$\left\{\begin{array}{l} Q = \{q_0, \ldots, q_8\} \\ \delta(q_j, a) = \begin{cases} q_{j+1} & \text{if } j \leq 7 \\ q_8 & \text{if } j = 8 \end{cases} \end{array}\right.$$

$F = \{q_5, q_7\}$

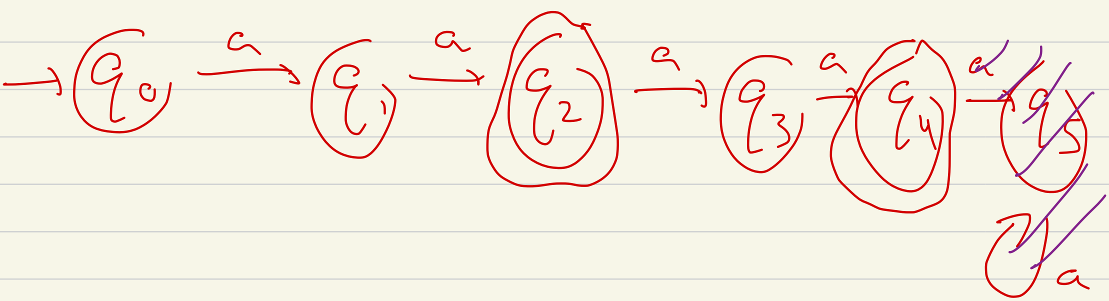
OR



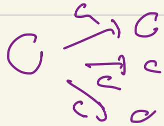
$\{a^2, a^4\} = L$



$L^* = \{a^2, a^4\}^*$

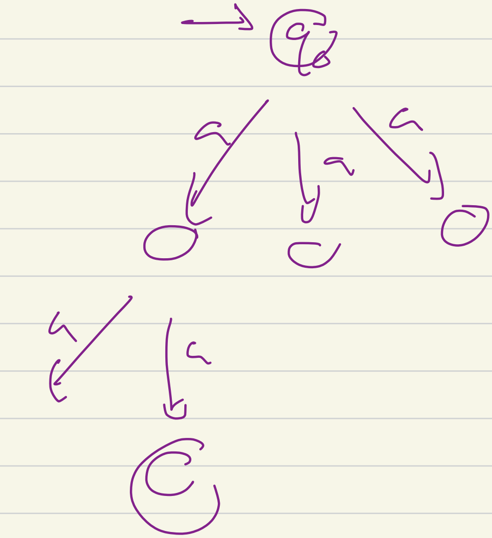


non-determinism!



no outgoing arrow

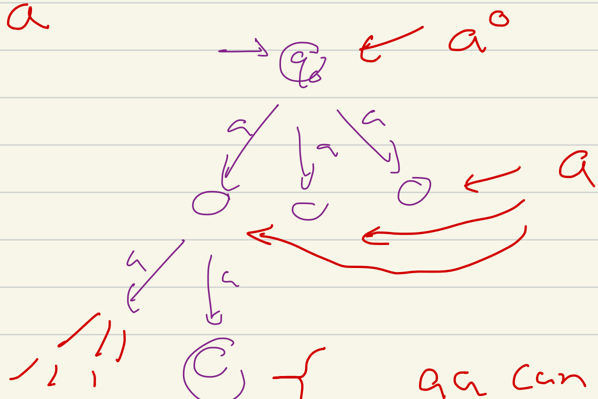
Conventions!



You can go to any state, and if there is at least one way to reach accepting state, then the algorithm accepts

Machine accept $aa = a^2$

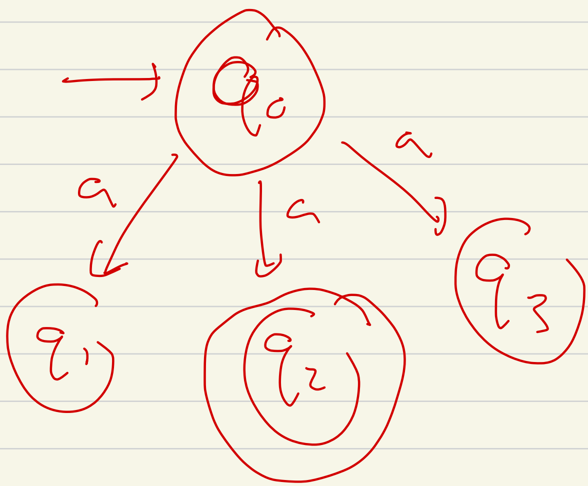
aa



aa can land here

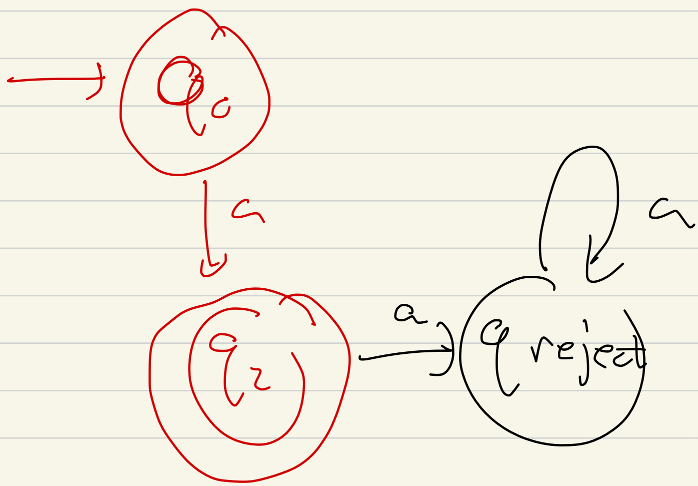
If there is no outgoing arrow, then we automatically reject, even in an accepting state

Clarification:



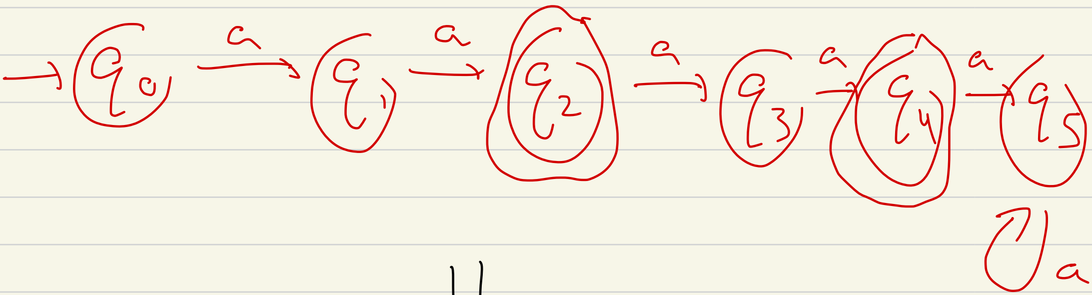
recognizes $\{a\}$

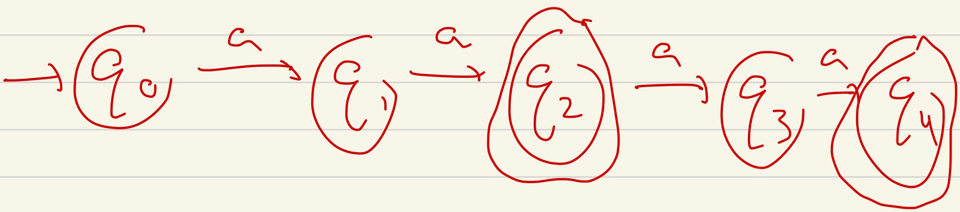
DFA:



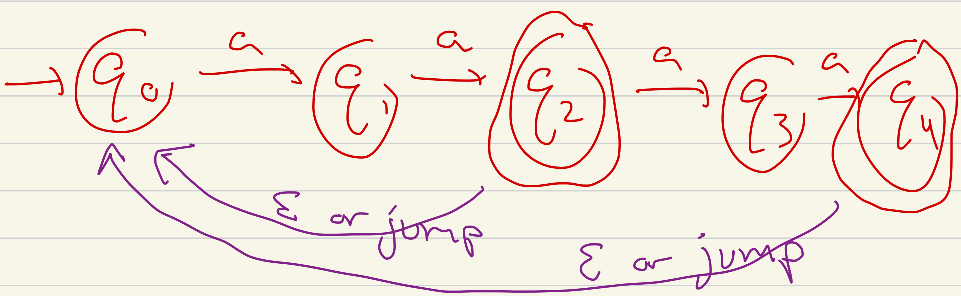
(Any NFA has a corresponding DFA)

NFA for $\{a^2, a^4\}$





$\{a^2, a^4\}^*$



Allow a "jump" in an NFA

NFA's!

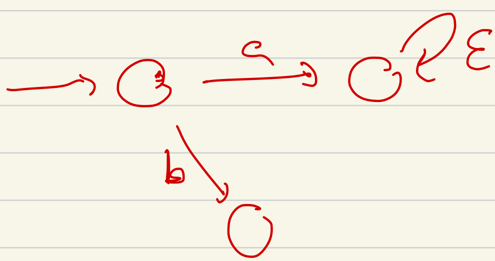
(1) out of a state, with a given symbol $\sigma \in \Sigma$, we can see

0, 1, 2, ... ways to go

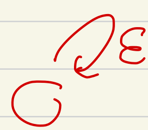
(2) there is an "ε" or "jump" arrow

that our machines allow.

Clarify: this is OK



but



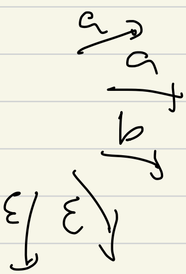
won't really help ...

Formally: an NFA is a

$(Q, \Sigma, \delta, q_0, F)$, all the

same as a DFA except:

$$\delta : Q \times \Sigma_\varepsilon \to \mathrm{Power}(Q)$$



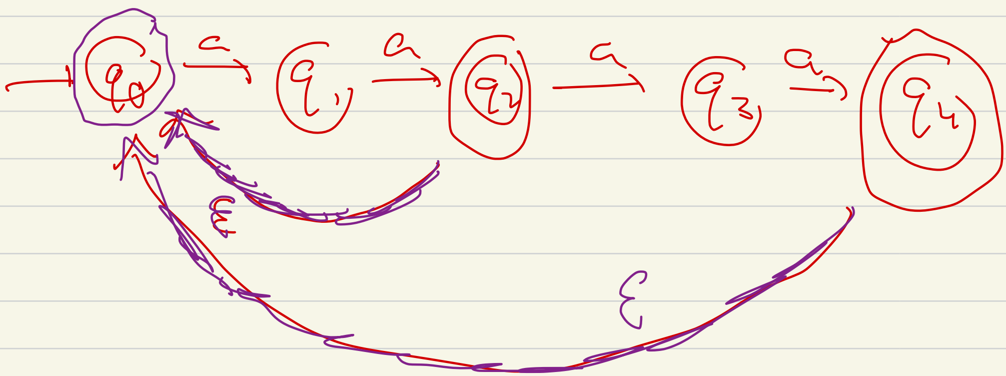
$\Sigma = \{a, b\}$

$$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$

Subject to convention above

([Sip], Ch 1, Section 2)

e.g.,



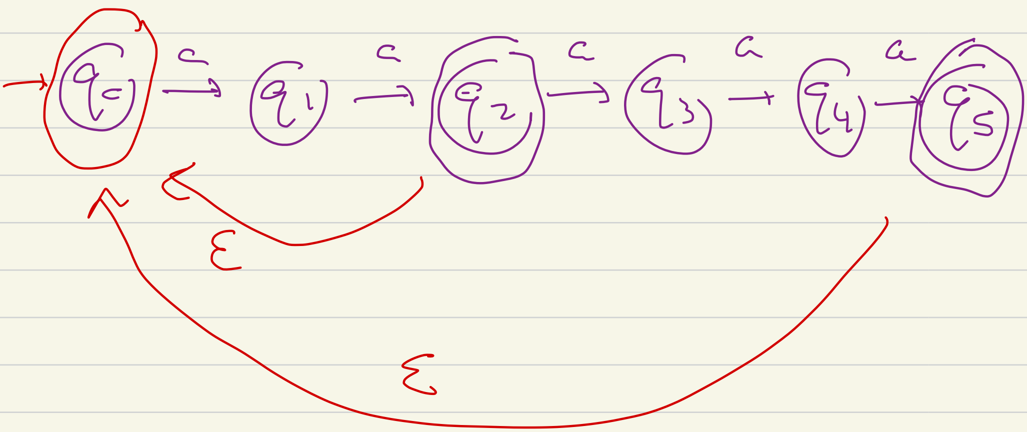
Claim: This accepts $\{a^2, a^4\}^*$

$$L^* = L^0 \cup L^1 \cup L^2 \cup \ldots$$

$$L^0 = \{\varepsilon\}$$

Rem: In the example above, we could delete q_3, q_4 and get the same language...

But $\{a^2, a^5\}^*$



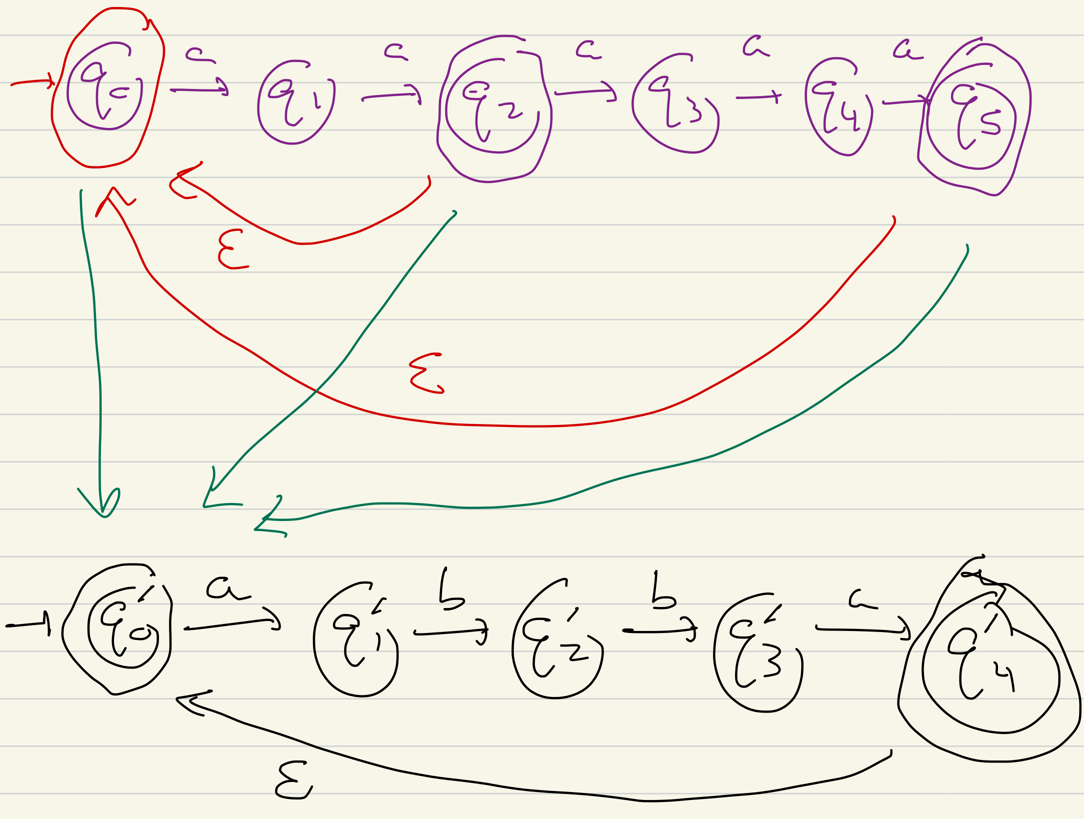
If you want to, you could remove all jumps (ε-arrows), but conceptually the jumps make things easier

$\Sigma = \{a, b, c\}$

What about

$\{a^2, a^5\}^* \circ \{abba\}^*$

concatenation



Implement:

$$L \to L^*$$

(1) make q_0 accepting

(2) allow ε jumps from any accepting state to q_0

Implement

$$L_1, L_2 \to L_1 \circ L_2$$

(1) Write machine for L_1 first, then
" " L_2

(2) We introduce jumps from all accepting states of L_1 to q_0' initial state for L_2

Next time:

any NFA has corresponding DFA