CPSC 421/501 Oct 3, 2025

Q: Can you run DFA's "in parallel"?

E.g. $\Sigma = \{a, b\}$

$$L_1 = \left\{ s \in \Sigma \;\middle|\; \begin{array}{c} \text{\# of a's in} \\ s \text{ is odd} \end{array} \right\}$$

$$L_2 = \left\{ s \in \Sigma \;\middle|\; \begin{array}{c} \text{\# of b's in} \\ s \text{ is div by 3} \end{array} \right\}$$

DFA for: $L_1 \cup L_2$? $L_1 \cap L_2$?

$L_1 \setminus L_2$?

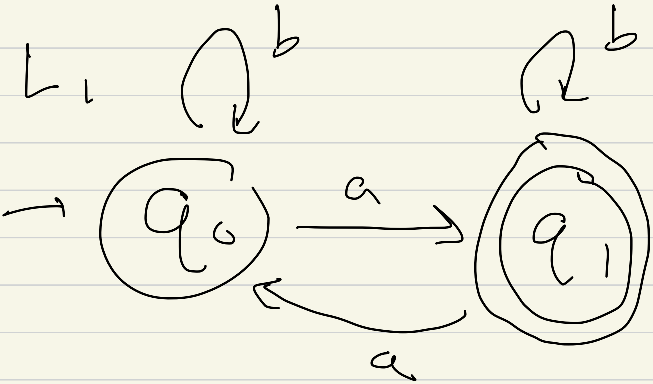
Next week: same question,

but also L reg $\Rightarrow L^*$ reg

via NFA <u>non-deterministic</u>

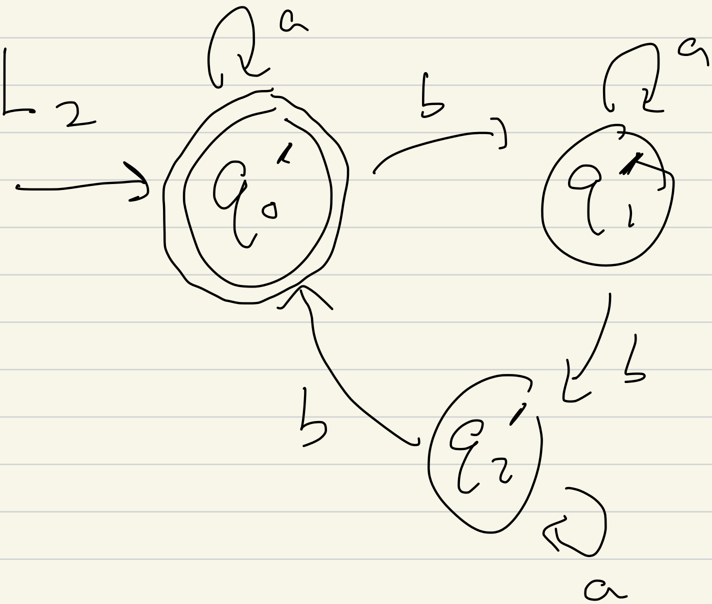
finite automata

L_1



like $\Sigma = \{a\}$
but ignore
b's

L_2



b's div
by 3

ignore
a's

Is $L_1 \cap L_2 = \{ s \mid$ both blah$_1$ and blah$_2 \}$

is $L_1 \cup L_2 = \{ s \mid$ at least one of blah$_1$ and blah$_2 \}$

$M_1 = (Q, \Sigma, \delta, q_0, F)$ recognizes L_1

$M_2 = (Q', \Sigma, \delta', q_0', F')$ recognizes L_2

Set of states:

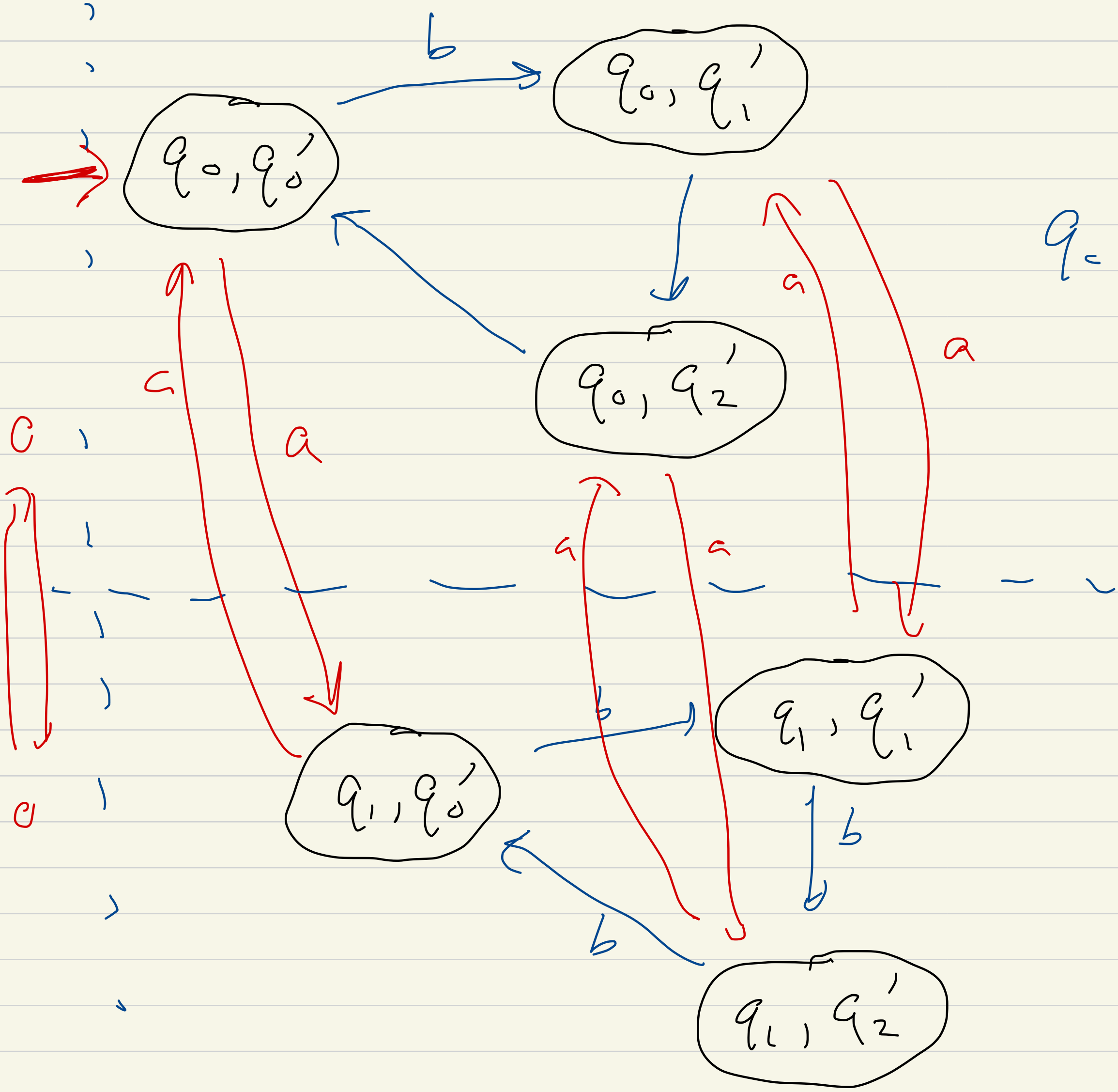
$$M = \left(Q \times Q', \Sigma, \delta'', (q_0, q_0'), F''\right)$$

where

for $L_1 \cup L_2$, F'' ...

for $L_1 \cap L_2$, F'' ...

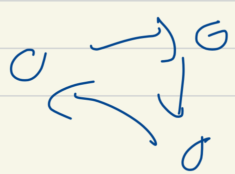
" $L_1 \setminus L_2$, F'' --



m_1

q_1

m_2



$$\delta'' : Q \times Q' \times \Sigma \longrightarrow Q \times Q'$$

$$\delta''(q, q', \sigma) = (\delta(q, \sigma), \delta'(q', \sigma))$$

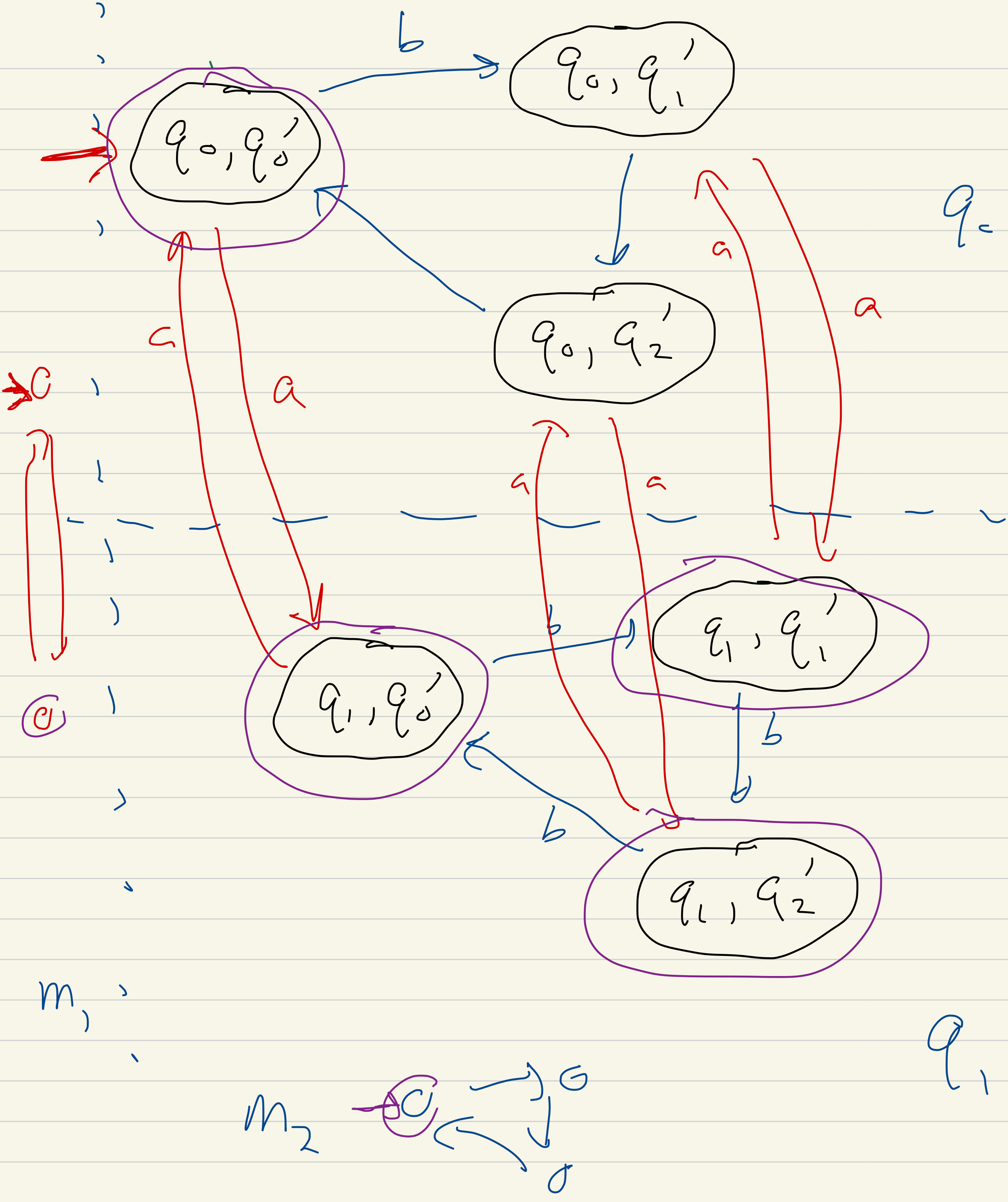
Recognize:

$L_1 \cup L_2$:

$$F'' =$$

$$F \times Q' \cup Q \times F'$$

\# a's is odd, \# b's is div by 3



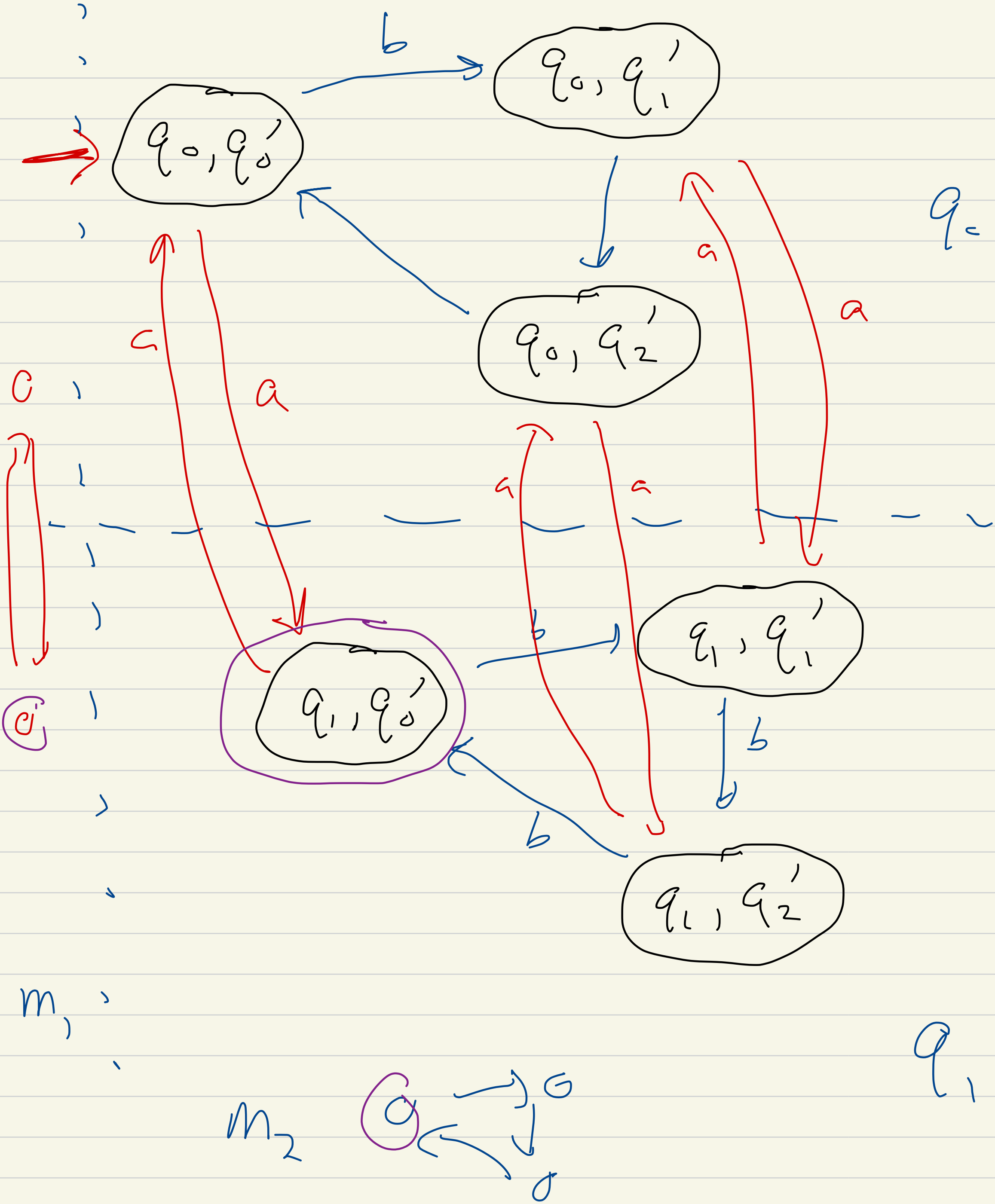
q_0, q_0'
q_0, q_1'
q_0, q_2'
q_1, q_0'
q_1, q_1'
q_1, q_2'
a
b
q_0
q_1
m_1
m_2

$L_1 \cap L_2$

either M_1 accepts AND

M_2 accepts

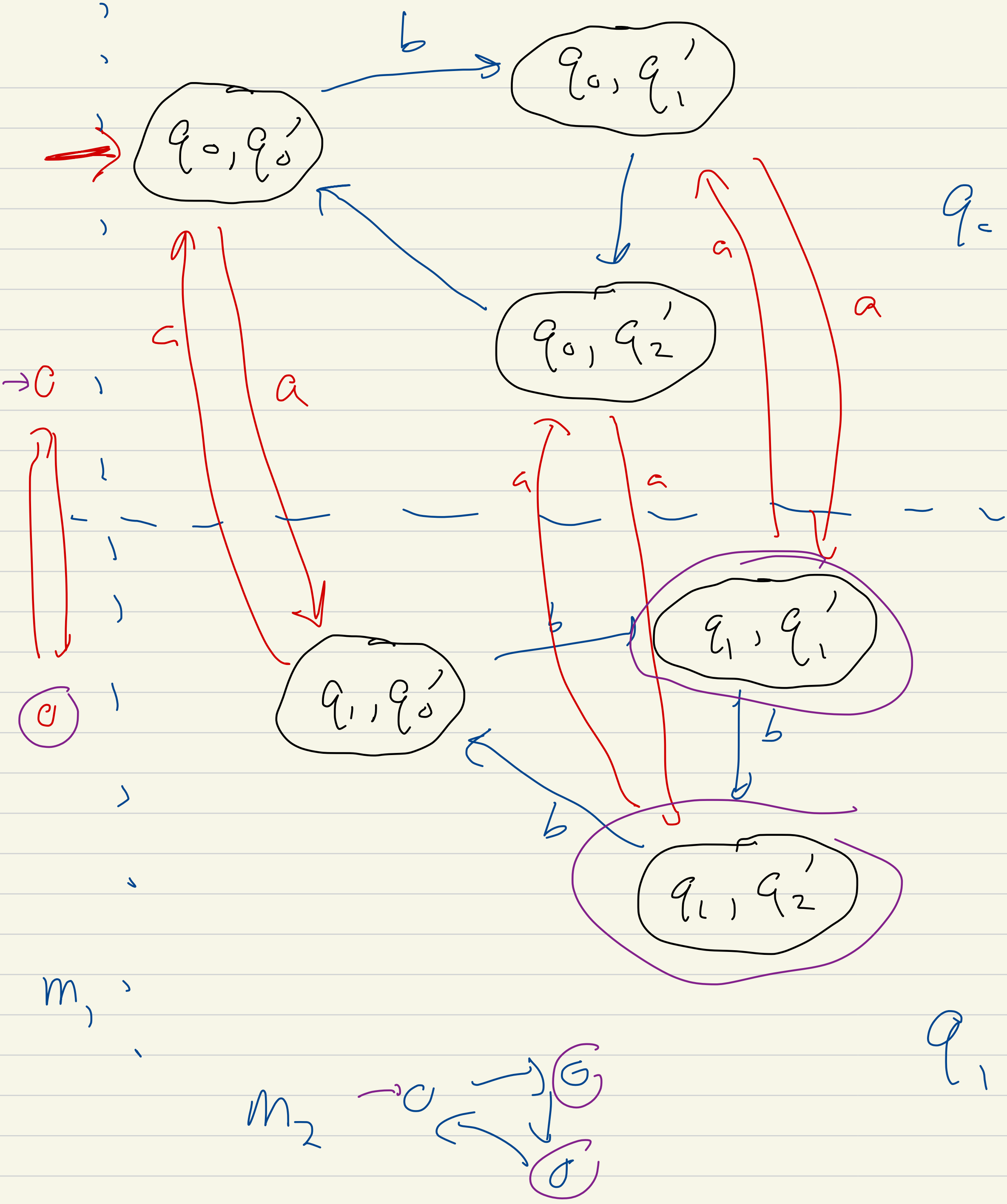
$F'' = F \times F'$


q0, q0'
q0, q1'
q0, q2'
q1, q0'
q1, q1'
q1, q2'
a
b
q0
q1
m1
m2

Similarly

$L_1 \setminus L_2$

$L_1 \cap L_2^{comp}$


q0, q0'
q0, q1'
q0, q2'
q1, q0'
q1, q1'
q1, q2'
a
b
q0
q1
M1
M2

We know: if L_1 is regular, L_2 is regular

$$\Rightarrow \left.\begin{array}{l} L_1 \cap L_2 \\ L_1 \cup L_2 \\ L_1 \setminus L_2 \\ L_2 \setminus L_1 \end{array}\right\} \text{regular}$$

Hence

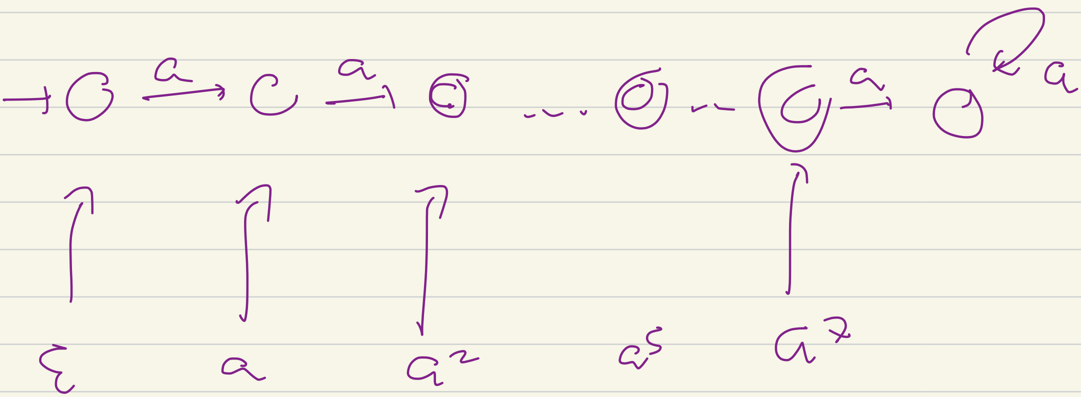
$L_1 \triangle L_2 = L_1 \setminus L_2 \cup L_2 \setminus L_1$ is regular

symmetric difference (XOR)

Is L_1^* regular ????

We've seen

$\{a^5, a^7\}^*$ is regular

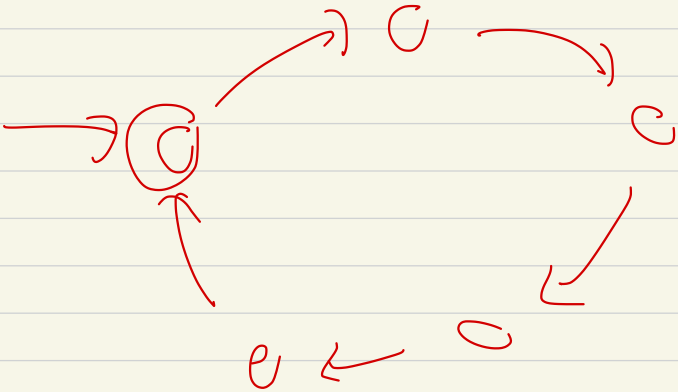


DFA $\{a^5, a^7\}$

$\rightsquigarrow$ Non-det FA

What about $L_1 \circ L_2$???

$\{a^5\}^*$



$\{a^7\}^*$

$$\{a^5\}^* \circ \{a^7\}^* = \{a^5, a^7\}^*$$

???

Here is a non-regular language

of $\{a, b\} = \Sigma$

$$\left\{ \left\{ a^n b^m a^n \;\middle|\; \begin{matrix} n \in \mathbb{N} \\ m \in \mathbb{N} \end{matrix} \right\} \right.$$

$$L = \left\{ a^p b^p \;\middle|\; p \text{ prime} \right\}$$

$L_1(a) L_2(b)$

$L = \{ a^p \} \cup \{ b^p \}$!

If

L is reg

so is

$L \cap \{a\}^*$ ...

Class ends