CPSC 421/501 Oct 1, 2025

Last time: DIV-BY-3 = $\{3, 6, 9, 12, 15, \ldots\}$

Formalism for DFA: (pl. DFAs)

$(Q, \Sigma, \delta, q_0, F)$

deterministic finite automaton (pl. automata)

Today: Regular and non-regular languages over $\Sigma = \{a\}$

If L_1, L_2 regular, so is $L_1 \cap L_2$, etc.

Non-regular languages over $\{a, b\}$

HW 4 due Monday, Oct 6

(" " revised Monday, Sept 29)

My office hours moved to

Monday afternoon

$$(Q, \Sigma, \delta, q_0, F) :$$

set of states — Q

alphabet — Σ

transition function — δ

$\delta : Q \times \Sigma \to Q$, q_0 = initial state

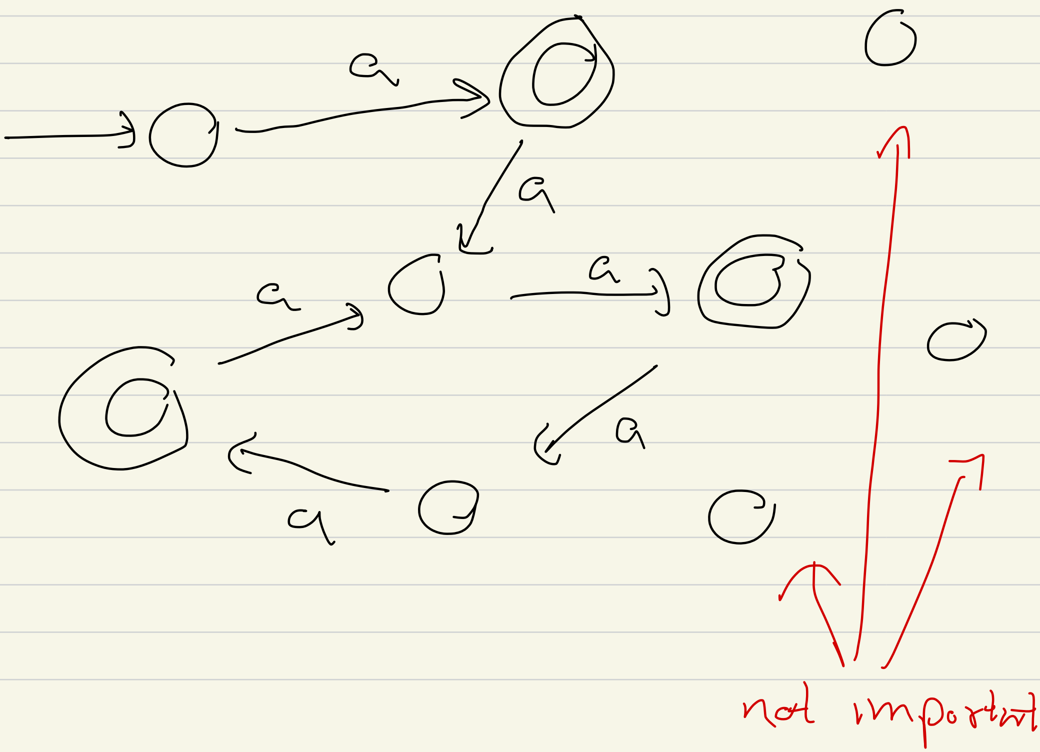
F = accepting (final) states

$\underbrace{\rightarrow \bigcirc}_{q_0}$

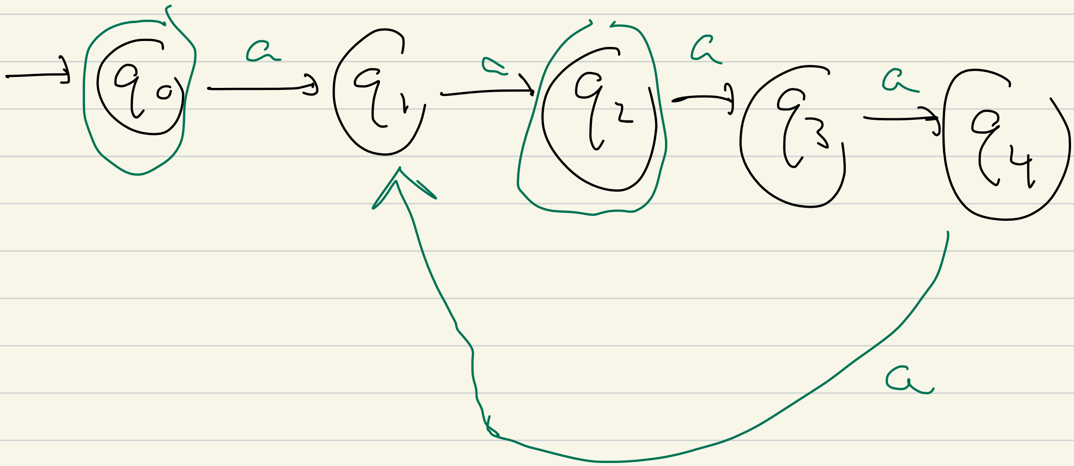
$\bigcirc$ – non-accepting

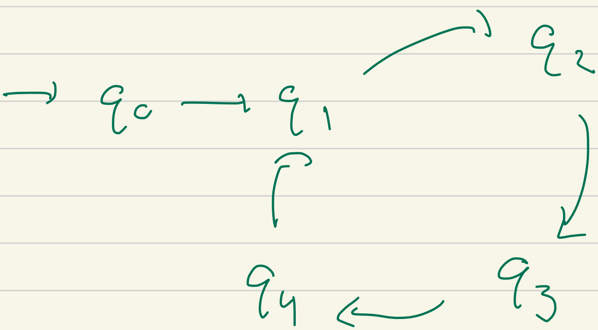
$\circledcirc$ = state in F

$\Sigma = \{a\}$

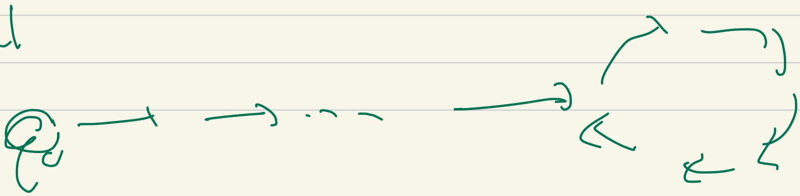


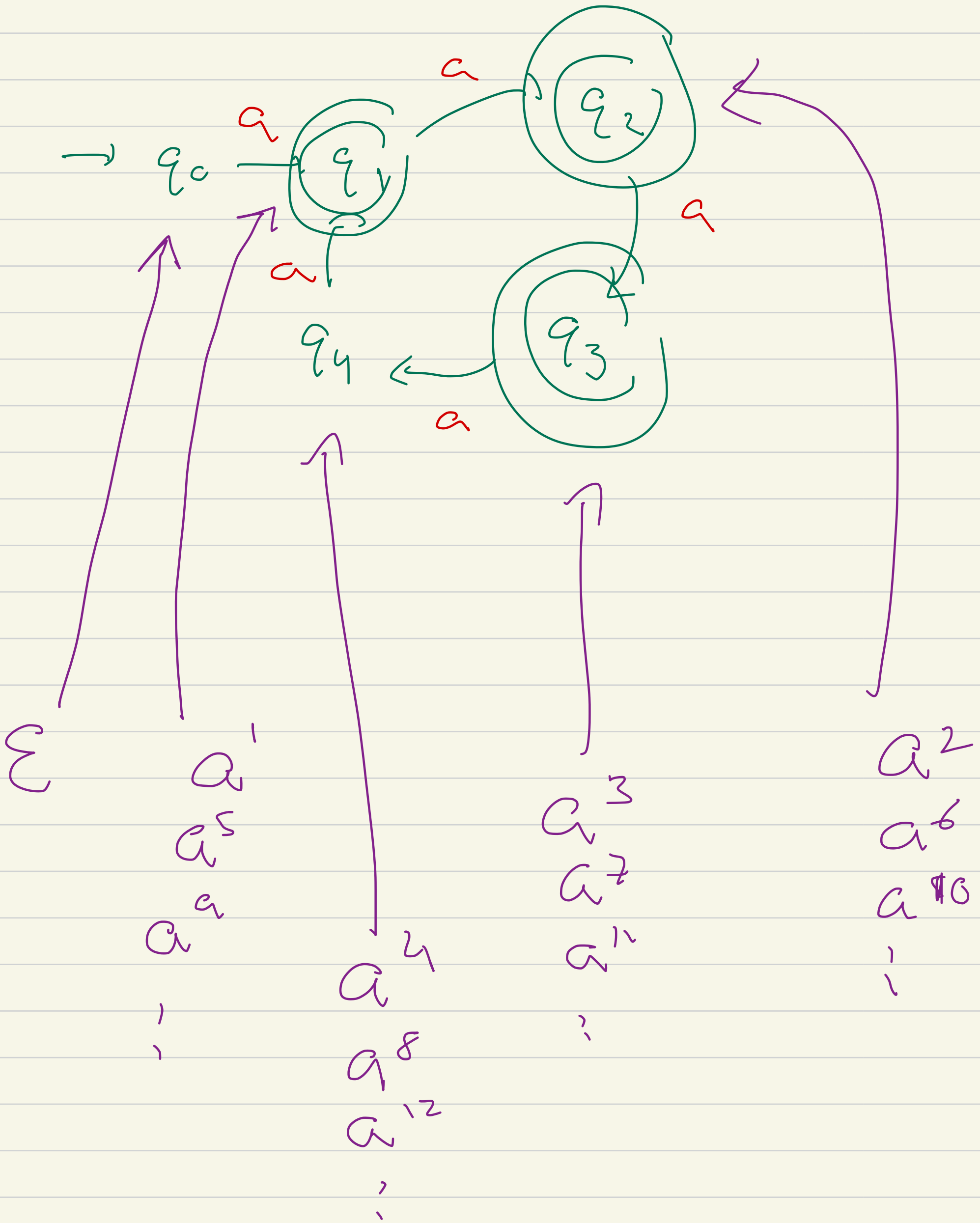
Any DFA over $\Sigma = \{a\}$





General




q0
q1
q2
q3
q4
a
a
a
a
a
ε
a^1
a^5
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a^4
a^8
a^12
a^3
a^7
a^11
a^2
a^6
a^10

A DFA, $M = (Q, \Sigma, \delta, q_0, F)$

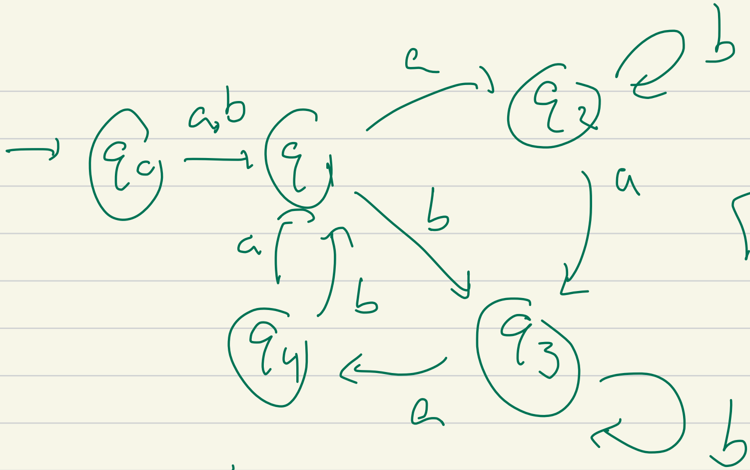
<u>recognizes</u> the language

$$L = \left\{ s \in \Sigma^* \;\middle|\; \begin{array}{l} \text{when } s \text{ is run on} \\ \text{the DFA, } s \text{ ends up in} \\ \text{a state in } F \end{array} \right\}$$

A language $L \subset \Sigma^*$

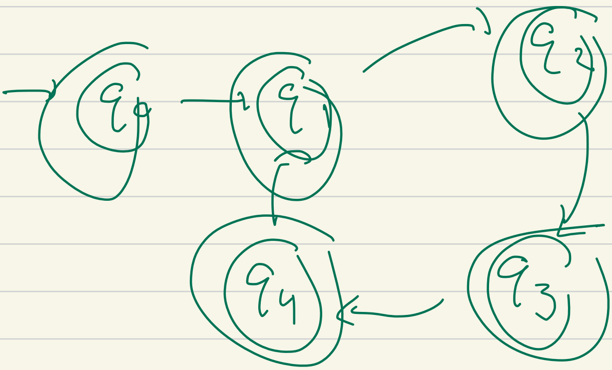
(any Σ) is called

- <u>regular</u> if L is <u>recognized</u> by some DFA
- non-regular if not (e.g. PALINDROME with $|\Sigma| \geq 2$)



$F = \emptyset$

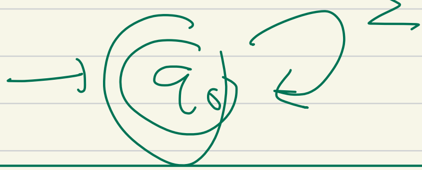
recognizes $\emptyset$



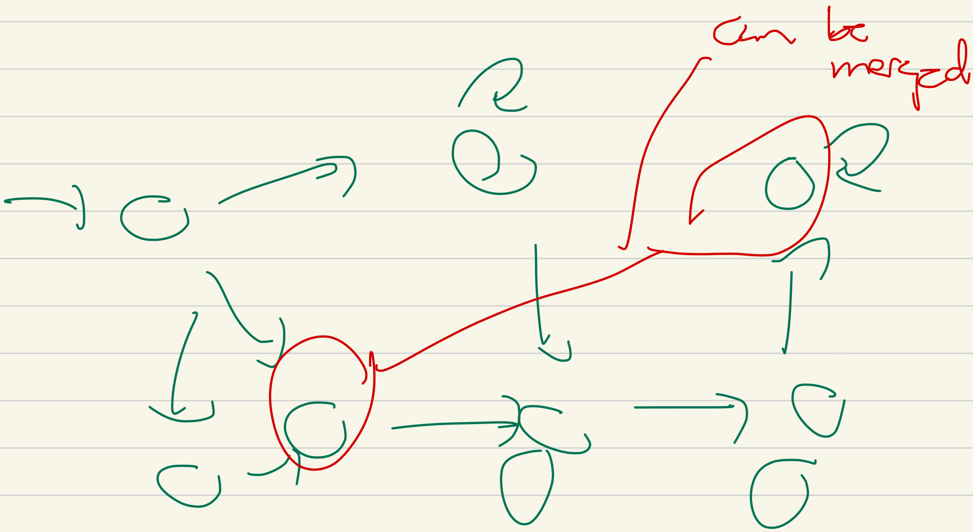
$F = Q$

recognizes Σ^*

but



Q: Given $L \subset \Sigma^*$ regular, what is the fewest # of states needed in a DFA that recognizes L ?

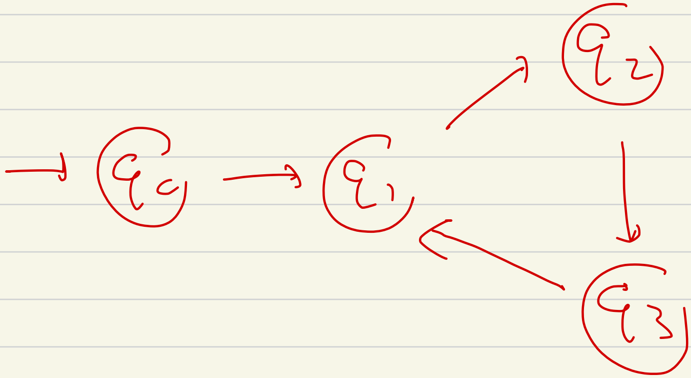


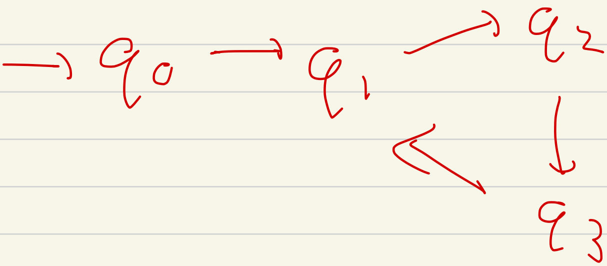
Say you "understand a DFA"

What if

Regular languages : $\emptyset$, Σ^*

$\Sigma = \{a\}$

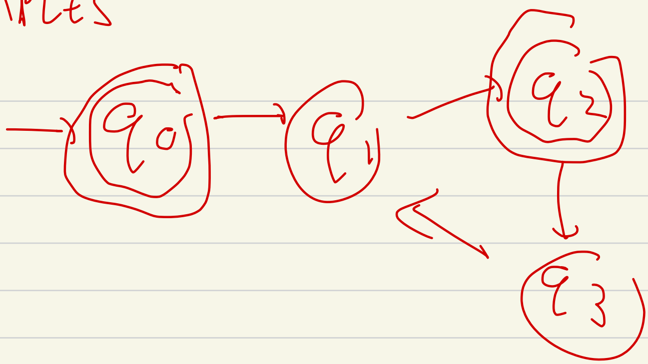




some are accepting

some are rejecting

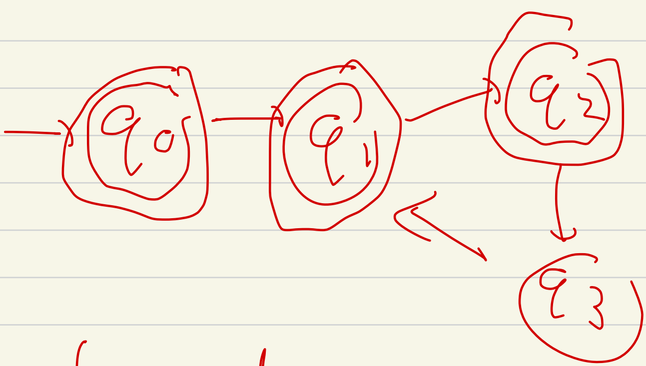
EXAMPLES



recognizes

$$L = \{ \varepsilon = a^0, a^2, a^5, a^8, a^{11}, \ldots \}$$

$$= \left\{ a^m \;\middle|\; \begin{array}{l} m = 0 \\ \text{or } m \geq 2 \text{ and} \\ m \bmod 3 = 2 \end{array} \right\}$$



$$\left\{ a^m \;\middle|\; m \leq 2, \text{ or } \begin{array}{l} m \geq 3 \text{ and} \\ m \bmod 3 = 1, 2 \end{array} \right\}$$

$$= \{a^0\} \cup \{a^n \mid n \text{ not div by } 3\}$$

Consider

$$L = \{a^p \mid p \text{ prime, i.e. } p = 2, 3, 5, 7, 11, 13, \dots\}$$

- intuition
- claim: there is no known pattern
- consider: we know there are infinitely many primes. Let $n \in \mathbb{N}$, consider

$$n!+2, \; n!+3, \; \dots, \; n!+n$$

$(n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n)$

$n! + 2$ divis by 2

$n! + 3$ divis by 3

so on

$L \subset \{a\}^*$, L is infinite,

but

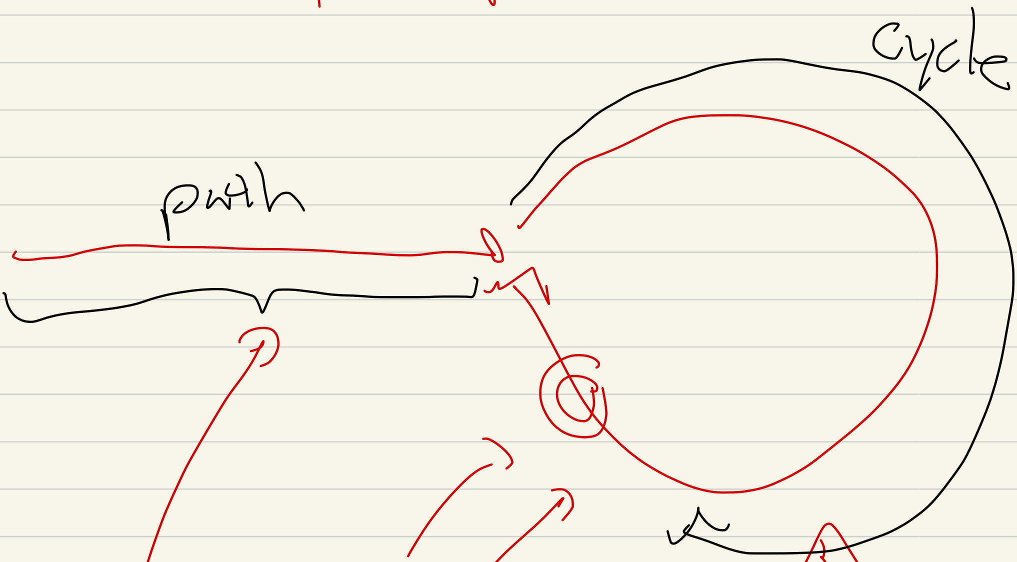
$L = \{ a^{m_1}, a^{m_2}, a^{m_3}, \ldots \}$

$m_1 < m_2 < m_3, \ldots$

we know

$m_{i+1} - m_i$ "gap from a^{m_i} to $a^{m_{i+1}}$"

this gap, $m_{i+1} - m_i$ can be arbitrarily large, so



some state has to accept

length of the path

"so gap is bounded"

length of the cycle

gaps $1, 2, 3, 4, \ldots$

$$a^{1}, \quad a^{1+1}, \quad a^{1+1+2}, \quad a^{1+1+2+3}, \ldots$$

(we don't care what is a formula for

$$1 + 1 + 2 + 3 + 4 + \ldots + n$$
)