CPSC 421/501 Sept. 29, 2025

DIV-BY-3 = $\{3, 6, 9, 12, 15, \ldots\}$

Variants = ?

Formalism for DFA: (pl. DFAs)

$(Q, \Sigma, \delta, q_0, F)$

deterministic finite automaton
(pl. automata)

Examples of regular and (over $\Sigma = \{a\}$) non-regular languages

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NFA (non-deterministic), $\{a^5, a^6\}^*$

$$\text{DIV-BY-3} = \{3,6,9,12,15,\underbrace{\ldots}_{30}\}$$

$$\Sigma = \{0,1,2,\ldots,9\}$$

Question!

$$\text{DIV-BY-3-WITH-ZERO-OK} = \{0, 3,6,9,12,\ldots\}$$

$$\text{DIV-BY-3-LEADING-0's-OK} = \{0,3,6,9,00,03,06,09,\ldots\}$$

$$\text{DIV-BY-3-LEADING-0's-OK-}\varepsilon\text{-OK} = \{\varepsilon,0,3,6,9,00,03,06,09,\ldots\}$$

DIV-BY-3 = $\{3, 6, 9, 12, 15, \ldots\}$

input

111111 3 = s

$s = \sigma_1 \sigma_2 \ldots \sigma_\ell$

$\ell = |s| = \text{length}(s)$

Algorithm: Possible inputs:

- $111 \in L$ = DIV-BY-3
- $0111 \notin L$
- $\varepsilon \notin L$

"initial state"

$\rightarrow \bigcirc$, $\rightarrow \circledcirc$

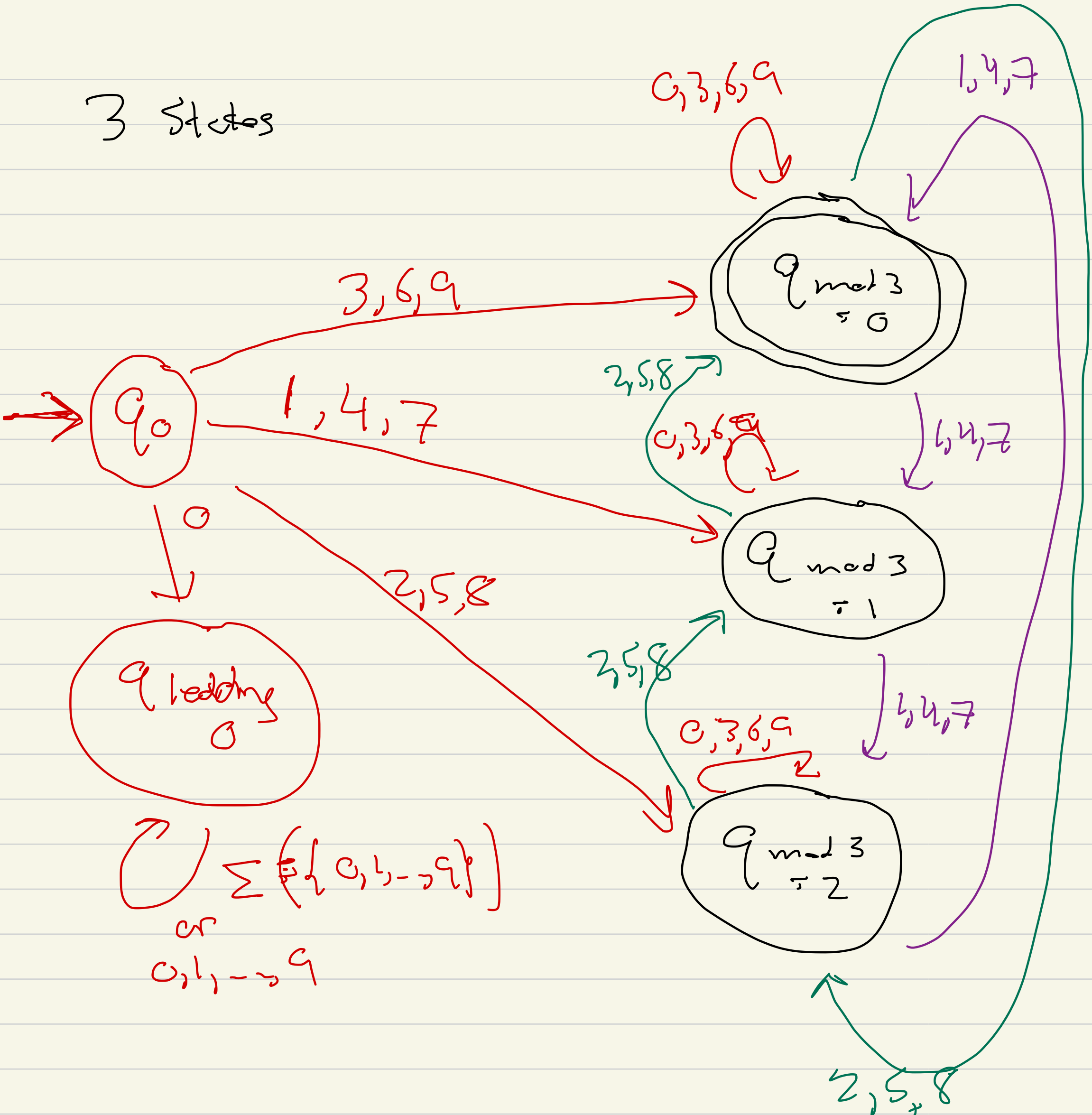
3 states:

so far, what we've seen, is div by 3

so far, what we've seen, mod 3 = 1

so far — —
— — mod 3 = 2

3 States



$q \xrightarrow{\sigma_1, \sigma_5} q'$ when we read σ_1, σ_5 in state q, move to q'

(q $\quad$ q ← we accept this input)

○ with no ◎ means we reject the input

1 1 1 3 2

1 1 1 3 2

end of string

stop / halt

0 3 6 9

stop

Set of states, Q, (finite set)

Initial state $q_0 \in Q$

$\delta : Q \times \Sigma \longrightarrow Q$ (transition function)

$\delta(q, \sigma)$ = the state you go to when you are in state q, and you read symbol $\sigma \in \Sigma$.

Can describe by (1) diagram

(2) list all values:

$\delta(q_0, 3) = q_{\text{mod } 3 = 0}$,

$\delta(q_0, 4) = q_{\text{mod } 3 = 1}$,

etc.

(3) table

	0	1	2 ...
q_0	$q_{\text{leading } 0}$		
$q_{\text{leading } 0}$	$q_{\text{leading } 0}$		
⋮			

F = final states

= accepting states. $F \subset Q$

$(Q, \Sigma, \delta, q_0, F)$ = DFA

Definition: A DFA (deterministic finite automaton) is a 5-tuple

$M = (Q, \Sigma, \delta, q_0, F)$ s.t.

- Q, Σ finite sets (state set, alphabet)
- $\delta : Q \times \Sigma \to Q$ (transition function)
- $q_0 \in Q$ initial state
- F set of (final) accepting states, $F \subset Q$

F can be empty

e.g., $\Sigma = \{a\}$

what is a DFA?