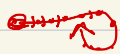
CPSC 421/501 Sept 26, 2025

[Sip] Michael Sipser, Intro. to the Theory of Computation

Chapter 1 ---

... except

(1) trivial construction of non-regular languages (eventual periodicity, )

(2) Myhill-Nerode

(3) Skip:

- Pumping Lemma
- DFA $\longrightarrow$ Regular Language

Formally:

DFA = deterministic finite automaton

$M = (Q, \Sigma, \delta, q_0, F)$

Q = states of M

Σ = alphabet of M

δ = transition function of M

q_0 = initial state of M

$F \subseteq Q$

= the final/accepting states of M

Examples: - DIV-BY-3

- $\Sigma^* \text{UBC} \Sigma^* = \Sigma^* \circ \text{UBC} \circ \Sigma^*$

Recall the <u>operations</u>

$L_1 \cup L_2$, $L_1 \cap L_2$, L^{comp}, $L_1 \setminus L_2$

"Review" the operations:

$L_1 \circ L_2 \stackrel{def}{=} ?$

$L^* \stackrel{def}{=} ?$

e.g. $\{a^2, a^9\}^*$

I will post which sections of Uncomputability in CPSC 421 we have covered so far.

DIV-BY-3

$\{a^{\ell_1}, a^{\ell_2}\}^*$

If S_1, S_2 are sets, then

$S_1 \cup S_2$, $S_1 \cap S_2$,

$S_1^{comp} = (\text{big thig, universe}) \setminus S_1$

typically Σ^*

Σ alphabet

- ASCII
- $\{a, b\}$
- $\{0, 1\}$

New operations, work with languages over an alphabet: $(L_1, L_2 \subset \Sigma^*)$

$$L_1 \circ L_2 = \left\{ s_1 \circ s_2 \;\middle|\; \begin{matrix} s_1 \in L_1 \\ s_2 \in L_2 \end{matrix} \right\}$$

$$L = L^1, L^2, L^3, L^4, \ldots$$

$$L^2 = L \circ L, \quad L^3 = L \circ L \circ L = LLL$$

$$= LL$$

$$L^* = \underset{?}{L^0} \cup \underbrace{L^1}_{L} \cup L^2 \cup L^3 \cup \ldots$$

Question! given L_1, L_2

can you get

L_1 ~~com~~ comme L_2 ~~def~~

$\{a\}$, $\{b, bb\}$

we want...

$$\{a, b, bb\} = L_1 \cup L_2$$

$$\{ab, abb, ba, bba\} = L_1 L_2 \cup L_2 L_1$$

L_1 XOR L_2 may be... = L

$\text{Cpsection}(L_1, L_2)$

$$= L_1 \cup L_2^{comp}$$

stuff in L_1 OR

not in L_2 $\leftarrow$ L_2^{comp}

$$L_1 \cap L_2^{comp}$$

$$L_1 \setminus L_2$$

$$S_1 \text{ XOR } S_2 = (S_1 \cup S_2) \setminus (S_1 \cap S_2)$$

$$= (S_1 \setminus S_2) \cup (S_2 \setminus S_1)$$

Example

$$\{a, abba\} \circ \{a, \varepsilon\}$$

$$= \left\{ \begin{array}{c} a \circ a, \\ a \circ \varepsilon, \\ abba \circ a, \\ abba \circ \varepsilon \end{array} \right\}$$

$$= \{ \underbrace{a \circ a}_{a^2}, a, abbaa, abba \}$$

Define: Σ alphabet, $\sigma \in \Sigma$,

$$\sigma^2 = \sigma \circ \sigma = \sigma\sigma, \quad \sigma^3 = \sigma\sigma\sigma$$

What is $L^0 = \{\varepsilon\}$

$\quad ? \; \emptyset$

$\quad ? \; \ldots$

$$L^5 = L^2 \circ L^3 = L^2 L^3$$

$LLLLL \qquad LL\ LLL$

$$L^0 L^2 = L^2$$

$$\varepsilon \circ S \stackrel{\text{def}}{=} S$$

(by convention
by exception
the empty set)

Hence

$$L^0 = \{\varepsilon\}$$

Σ = finite set

ε = unique string of length 0

$$\Sigma^3 \Longleftrightarrow \{1,2,3\} \to \Sigma$$

$$\Sigma^1 \Longleftrightarrow \{1\} \to \Sigma$$

$$\Sigma^0 \Longleftrightarrow \underbrace{\phi \to \Sigma}_{\text{exactly one } \varepsilon}$$

By convention, we don't allow $\varepsilon \in \Sigma$

$$\Sigma = \{ \{\emptyset\}, \emptyset, \{1,2,7\}, \{\{5\}\}, 5, \varepsilon \}$$

So ...

$$\underbrace{\{a^2, a^9\}}_{\substack{\text{set} \\ \text{language}}}^{*} \qquad \Sigma = \{a\}$$

$$= \{ \underbrace{\varepsilon}_{L^0}, \underbrace{a^2, a^9}_{L^1}, \underbrace{a^2a^2, a^2a^9, a^9a^2, a^9a^9}_{L^2}, \ldots \}$$

$$= \{ \varepsilon, a^2, a^9, a^4, a^{11}, a^{11}, a^{18} \}$$

$$\cup \{a^2, a^9\}^3 \cup \{a^2, a^9\}^4 \cup \dots$$

$$= \{ \varepsilon, a^2, a^4, a^6, a^8, a^9, a^{10}, a^{11}, \dots \}$$

not in here

missing: a a^3

$L = \{a^3, a^5\}^* \qquad \Sigma = \{a\}$

$= \{\varepsilon, a^3, a^5, a^6, a^8,$

$a^9, a^{10}, a^{11}, a^{12}, \ldots\}$

a^8, a^9, a^{10} in L

and $L = \{\ldots, a^3, \ldots\}$

...

$\{a^{10}, a^{25}\}^* = \{\varepsilon, a^{10}, \underbrace{a^{20}, a^{25}, \ldots}_{\text{and } a^{10}}$

higher multiples of 5