CPSC 421/501 Sept 24, 2025

- Friday begin regular languages

  [Sip], Chapter 1

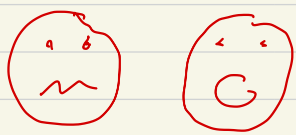
- Monday:

  LOOPING is unrecognizable

- Today:

  ACCEPTANCE, REJECTION,

  HALTING undecidable

- Also: Paradoxes 

Paradox emoji's

L is decidable if $\exists p$

s.t.

(1) $L = \text{LangRecBy}(p)$

(2) p is a <u>decider</u>

p is a <u>decider</u> if

$\forall i \in \text{ASCII}^*$,

either p accepts i, p rejects i,

i.e. p halts on every input

L is recognizable if $\exists p$ s.t.

(1) $L = \text{LangRecBy}(p)$, but not necc

2, i.e.

if $i \in L$, p accepts i

if $i \notin L$, p does not accept i,

i.e. $p \begin{Bmatrix} \text{rejects} \\ \text{loops} \end{Bmatrix}$ on i

If L is decidable, by p,

$\Rightarrow$

$$L^{comp} = ASCII^* \setminus L$$

is also decidable, namely by p_{new} given as:

p	p_{new} (from p)
return "yes"	"no"
return "yes"	"no"
return "no"	"yes"

Where $\quad$ compliment

L^{comp} (in the context of $ASCII^*$) refers to

$ASCII^* \setminus L$

where sets A, B

$$A \setminus B = \{ a \in A \mid a \notin B \}$$

set difference

p decides L : $\forall s \in ASCII^*$, if $s \in L$ then

if $s \notin L$ then

p

return "yes"

return "yes"

return "no"

p_{new} (from p)

"no"

"no"

"yes"

L is decidable

$\iff L^{comp}$ is decidable

Claim:

L_1	L_2	L_3
NOT-PYTHON-NP	ACCEPTANCE	REJECTION

L_4 = LOOPING ← unrecognizable

$L_4^{comp} = L_1 \cup L_2 \cup L_3$

cannot be decidable

$$\overbrace{L_1 \quad \cup \quad L_2 \quad \cup \quad L_3}^{\text{undecidable}}$$

NOT_PATH_NP ACCEPTANCE REJECTION

decidable (quickly)

Let's assume ACCEPTANCE is decidable, get contraction ---

Claim: ACCEPTANCE is decidable iff REJECTION is decidable

Proof: Say that

ACCEPTANCE is decidable

There is a program q s.t.

$$p \,\sigma_0\, i \xrightarrow[\text{to } q]{\text{as input}} q \text{ says "yes"}$$

iff

$p \,\sigma_0\, i \in$ ACCEPTANCE

iff

p accepts i

(q is a decider)

So, given

$\hat{p}\ \sigma_0\ \hat{i}$ ------ can we tell

if $\hat{p}$ on input $\hat{i}$ rejects?

$\hat{p}$:

return "no" → yes

return "yes" → no

return "yes" → no

$\longrightarrow \hat{p}_{new}$

Feed $\hat{p}_{new}\ \sigma_0\ \hat{i}$ to decider for ACCEPTANCE

Now:

if ACCEPTANCE is decidable,

also REJECTION " "

(And NOTaPYTH-INPUT " ")

If L_1, L_2, L_3 are each decidable,

then (claim)

$L_1 \cup L_2 \cup L_3$ is "

why? run deciders for L_1, L_2, L_3

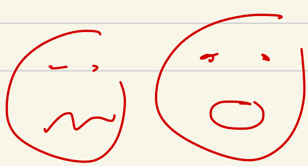
in parallel (or sequentially)

Look at the phrase

"the smallest positive integer not described by an English phrase of fewer than fifty words"

(say 1721493 … 172025)

due to Russell most likely



"There is a person who writes about and only about each person who doesn't write about themself."