

CPSC 421/501

Sept 19, 2025

- LOOPING is unrecognizable
 - ACCEPTANCE, REJECTION, HALTING are undecidable
 - NON-PYTHON-INPUT is decidable
-

$\exists c \in \text{ASCII}$ character that
no valid Python program
contains

Given $s \in \text{ASCII}^*$, s lies in exactly one of these sets

① $\{ p \sigma_0 i \mid p \text{ is a valid Python program that accepts } i \text{ (it encounters a return "yes")} \}$
 $= \text{ACCEPTANCE} \subset \text{ASCII}^*$

REJECTION =

② $\{ p \sigma_0 i \mid \begin{array}{l} \text{---} \quad \text{---} \quad \text{---} \\ \text{---} \quad \text{rejed} \quad \text{---} \\ \text{---} \quad \text{---} \quad \text{return "no"} \end{array} \}$

③ $\text{LOOPING} = \left\{ p \sigma_0 i \mid \begin{array}{l} p \in \text{VALID-PYTHON-PROG} \\ \text{and} \\ p \sigma_0 i \notin \text{ACCEPTANCE} \\ \text{REJECTION} \end{array} \right\}$

④ $\{ s \mid s \text{ is not of the form } p\sigma_i \text{ s.t. } p \in \text{VALID-PYTHON PROGRAM} \}$

HALTING = ACCEPTANCE $\cup$ REJECTIONS

$T = \{ p \in \text{ASCII}^* \mid p \notin \text{LangRecBy}(p) \}$

is not recognizable.

Decidable vs. recognizable

We say that

$p \in \text{VALID-PYTHON-PROGRAM}$

is a decider if it "halts on
all inputs", i.e.

$\forall i \in \text{ASCII}^*$

$p \sigma_i \in \text{HALTING}$

$\Rightarrow \text{ACCEPTANCE} \cup \text{REJECTION}$

i.e. p on input i always

reaches $\begin{cases} \text{return "yes"} \\ \text{return "no"} \end{cases}$

We say $L \subseteq \text{ASCII}^*$ is

decidable if $L = \text{LangRecBy}(d)$

for some d that is a decider.

Q: $d = "421hi2025"$

Voted: this is not a decider

So $p \notin \text{VALID-PYTHON-PROGRAM}$

then, by definition, p is not a
decider. (Want really matter)

We say $L \subseteq \text{ASCII}^*$ is

recognizable if $L = \text{LangRecBy}(p)$

for some $p \in \text{ASCII}^*$, i.e.

if $i \in L$, then $p \sigma_0 i \in \text{ACCEPTANCE}$
 p accepts i

$i \notin L$, then $p \sigma_0 i \notin \text{ACCEPTANCE}$,
i.e. p does not accept i ,

i.e. p either rejects or "loops"

e.g. ACCEPTANCE is recognizable
 HALTING " "

"debugger" "universal Python program"

Claim:

LOOPING is undecidable,

i.e. LOOPING is not in the image
of LangRecBy:

ASCII*	{	NOT-PYTHON-INPUT	L_1
		ACCEPTANCE	L_2
		REJECTION	L_3
		LOOPING	L_4

If L_1, L_2, L_3, L_4 is a partition of
ASCII*, then

L_1, L_2, L_3, L_4 are decidable

Pf:

Some program

P_1 recognizes L_1

P_2 " L_2

P_3 " L_3

P_4 " L_4

Given input $i \in \text{ASCII}^*$, and
you could ... run in parallel

$p \sigma_i$
 $\left\{ \begin{array}{l} \text{NOT-VALID} \\ \text{ACCEPT} \\ \text{REJECT} \\ \text{LOOPS} \end{array} \right.$

$$T = \{ p \mid p \notin \text{LangRecBy}(p) \}$$

is not recognizable

=

Claim: LOOPING is not

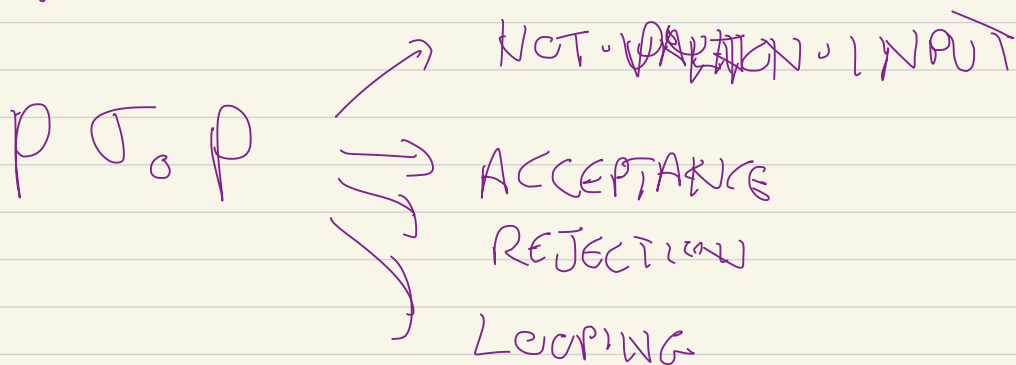
recognizable

Pf:

When is $p \notin \text{LangRecBy}(p)$?

When is $p \in \text{LangRecB}(p)$?

Ans:



and ---

CLASS ENDS