

CPSC 421/501

Sept. 15, 2025

- Language Recognized By :

$\underbrace{\text{ASCII}^*}_{\text{A finite length ASCII string}} \rightarrow \text{Power}(\text{ASCII}^*)$

$\underbrace{\text{A finite length ASCII string}}_{\text{interpret as a Python (Javascript, etc.) program}}$

On input
 $i \in \text{ASCII}^*$,
program says

either $\begin{cases} \underline{\text{yes}} \\ \underline{\text{no}} \end{cases}$

or "loops"

i.e. not yes, not no

This is a comment

#

```
def isDivByThree( ):
```

```
    print("Give me a positive integer.")
```

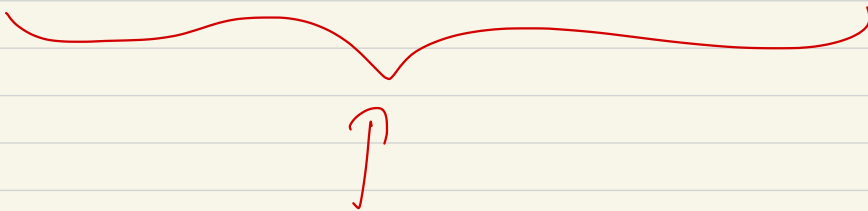
```
    i = input("Your input: ")
```

```
    while ( i > 0 ) : i = i - 3
```

```
    if ( i == 0 ) :
```

```
        return ("yes")
```

```
    return ("no")
```



Just some ASCII string

DIV-BY-3 over $\Sigma = \{0, 1, \dots, 9\}$

? { $\{3, 6, 9, 12, 15, \dots\}$
 $\{0, 3, 6, 9, \dots\}$
 $\{0, 3, 6, 9, 00, 03, \dots\}$
etc.

All caps for LANGUAGES

You have to carefully specify
what you mean

Python / C++ / APL / ...

Conventions!

A "program" takes an input

$i \in \text{ASCII}^*$

$\text{ASCII}^0 \cup \text{ASCII}^1 \cup \text{ASCII}^2 \cup \dots$

{ set of finite length ASCII strings }

Result: "yes"

"no"

"loop" — anything else

```
# This is a comment
```

```
#
```

```
def isDivByThree( ):
```

```
    print("Give me a positive integer.")
```

```
    i = input("Your input: ")
```

```
    while ( i > 0 ) : i = i - 3
```

```
    if ( i == 0 ) :
```

```
        return("yes")
```

```
    return("no")
```

Index: input 7

$i = 7, i = 4, i = 1, i = -2$

input 123123, $i = 123123$, $i = 123120$

$i = 123117, \dots$

$\dots i = 0$

Problem:

On input: "hi"

isDivByThree()

gives:

Traceback (most recent call last)

...

TypeError ...

This is a comment

#

```
def isDivByThree( ):
```

```
    print( "Give me a positive integer.")
```

```
    i = input( "Your input: " )
```

```
    while ( i > 0 ) : i = i - 3
```

```
    if ( i == 0 ) :
```

```
        return ( "yes" )
```

```
    if ( i == -1 or i == -2 )
```

```
        return ( "no" )
```

Input i = -3 :

i = -6, i = -9, i = -12, ...
does not halt !!

This is a comment

#

```
def isDivByThree( ):
```

```
    print("Give me a positive integer.")
```

```
    i = input("Your input: ")
```

```
    while ( i != 0 or i != -1  
            or i != -2 )
```

```
        i = i - 3
```

```
    if ( i == 0 ):
```

```
        return("yes")
```

```
    if ( i == -1 or i == -2 )
```

```
        return("no")
```

return "yes"

return "no"

{ other: TypeError, never halt ...
↓
"loops"

Seq program $p \in \text{ASCII}^*$

{ accepts i (as an input) if
running p on i returns yes.

{ rejects i
... .. no

for each $p \in \text{ASCII}^*$, the
language recognized by p ,

$\text{LanguageRecBy}(p)$, is defined
to be

$$\{ i \in \text{ASCII}^* \mid p \text{ accepts } i \}$$

$\subset \text{ASCII}^*$, Hence

$\text{LanguageRecBy} : \text{ASCII}^*$

$\longrightarrow \text{Power}(\text{ASCII}^*)$

LanguageRecBy(p)

def $\{ i \in \text{ASCII}^* \mid p \text{ returns}$
"yes" on input $i \}$

By Cantor's Theorem

$$T = \left\{ s \in \text{ASCII}^* \mid \right. \\ \left. s \notin \text{LanguageRecBy}(s) \right\}$$

is not in the image of

$\text{LanguageRecBy} \quad (!)$,

(i.e. $\nexists$ (there does not exist
 $t \in \text{ASCII}^*$ s.t. $T = \text{LanguageRecBy}(t)$)

Note: how can you, given,

$S \in \text{ASCII}^*$, tell if an

input S , the program S

says yes ??

Is there an algorithm that

given S ,

and given that S says yes

or no on S will tell you

so in a finite amount of time.

Ans: Yes, you could run "a debugging

program" or

"universal Python machine"

simulates what σ would do

on any input

Course ends

$(1, 2) (1, 2) (1, 2) (1, 2) (1, 2) ()$

$11 \rightarrow 1$

$12 \rightarrow 2$

$21 \rightarrow 3$

$22 \rightarrow 4$