

CPSC 421/501

Sept. 12, 2025

Uncomputability in CPSC 421/501

- Diagonalization + Partial Information + Yes/No Table
e.g. Leviathan Wakes, The Lathe of Heaven, etc.

- $\mathbb{R}$ is uncountable (what does an uncountable set "look like")

- Language Recognized By :

$\text{ASCII}^* \rightarrow \text{Power}(\text{ASCII}^*)$

We know

Ursula LeGuin has written:

The Lathe of Heaven

The Dispossessed (Oppenheimer)

The Left Hand of Darkness

Daniel Abraham & Ty Franck

("James S.A. Corey") cowrote

Leviathan Wakes

(+ 8 other books in the series)

$S = \{ \text{Ursula, Daniel, Ty} \}$

$\text{Books} = \{ \text{Lathe, Dispossessed, ...} \}$

QR !

Movies:

{ Oppenheimer,

Barbie,

2001: A Space Odyssey,

Encounters at the End of the World }

$$| \text{Movies} | = 4$$

Cartier's Thm: $f: S \rightarrow \text{Power}(S)$

Really, ... $g: A \rightarrow \text{Power}(B)$

where $A \neq B$

What does an uncountably infinite set look like?

- Power ($\mathbb{N}$)
- Power (ASCII^*)
- $\mathbb{R}$

Thm: The real numbers, $\mathbb{R}$, is an ~~uncountable~~ uncountable set.

Caution: $0.9999\ldots = 1$

Look at real numbers on $[0, 1)$

$0 \leq r < 1$. Say that there is
a bijection

$$1 \mapsto r_1$$

$$2 \mapsto r_2$$

$$3 \mapsto r_3$$

$\vdots$

i.e., say $f: \mathbb{N} \rightarrow [0, 1) \subset \mathbb{R}$

then $\text{image}(f)$ is not all of $[0, 1)$

$$0.129999\dots = 0.13$$

$$r_1 \quad 0.\textcolor{violet}{1}23198\dots$$

$$r_2 \quad 0.13129$$

$$r_3 \quad 0.78\textcolor{violet}{2}1$$

2nd digit

1st digit 3rd digit

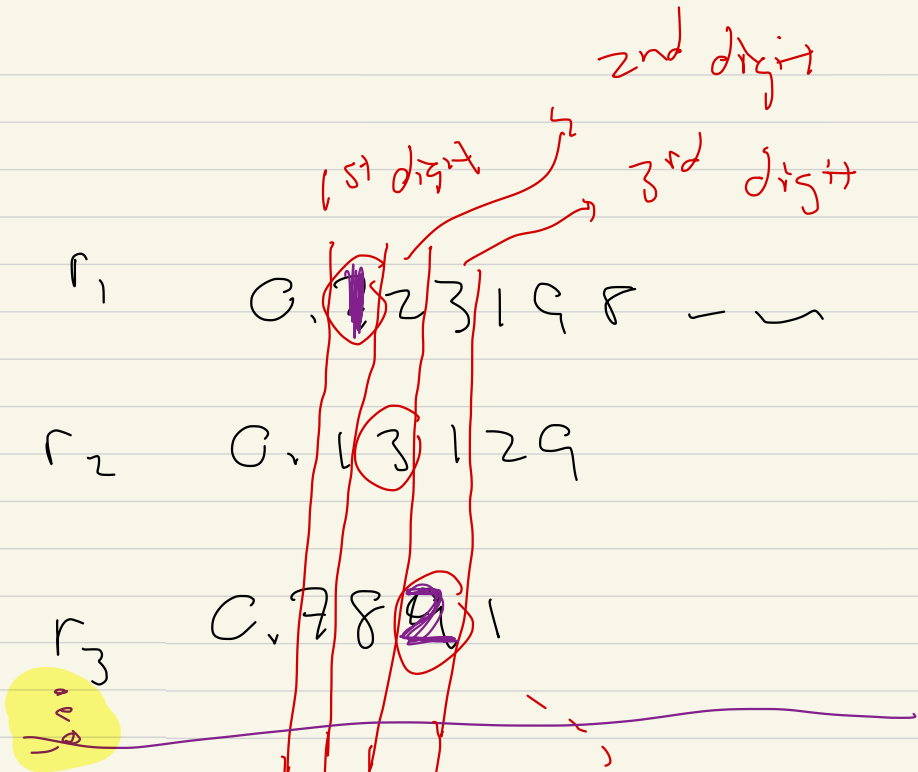
$$r_1 \quad 0.\textcircled{\textcolor{violet}{1}}23198\dots$$

$$r_2 \quad 0.1\textcircled{3}129$$

$$r_3 \quad 0.78\textcircled{\textcolor{violet}{2}}1$$

$\vdots$

r want its digits to be 1's, 2's



r 0. 2 1 1 - - -



$r \neq r_3$ etc.

$r \neq r_1$ $r \neq r_2$

In this way, we have built an

$$r, \quad 0 \leq r < 1$$

$$r \neq r_i \quad \text{for } i = 1, 2, 3, \dots$$

$$\text{So } r \notin \text{Image}(f)$$

So f is not surjective



$$r_1 = 0.1000 \dots \quad \frac{1}{10}$$

$$r_2 = 0.142857 \quad \frac{1}{7}$$

$$r_3 = \dots \quad \frac{6}{\pi^2}$$

Now:

$$\{r \in \mathbb{R} \mid 0 \leq r < 1\}$$

is uncountable

Now:

HW: $S \rightarrow T$ is an inject

(e.g. $[0,1) \xrightarrow{\text{inclusion}} \mathbb{R}$)

and S is uncountable,

then T " " " .

$\mathbb{R}$ vs. $\text{Power}(\mathbb{R})$



bijection?

surjection?

$\text{Power}(\mathbb{R})$ vs. $\text{Power}(\text{Power}(\mathbb{R}))$

OK:

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Encounters at the End of the World }

| Movies | = 4

REMARKS

Has
you?
seen?

Gpp

Barbie

2001

Encounters

A.K.

Yes

?

?

?

HZ

?

Yes

?

?

MY

?

?

?

No

AA.

?

?

Yes

?

XX

No?

No?

No

Yes?

No

No

No

Yes

[, , , Encounters]

