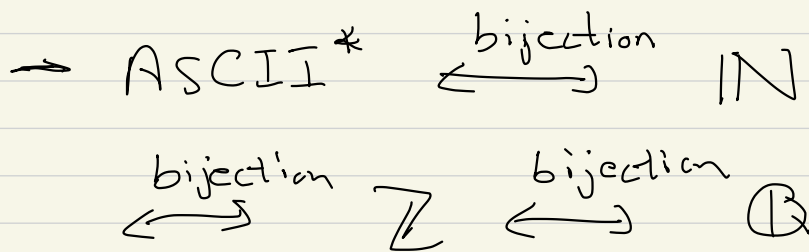


CPSC 421/501

Sept 10, 2025

→ Bad/good news: any finite document (e.g. your PhD thesis) is merely an element of ASCII^*



Countably infinite

— Countable $\leftrightarrow$ Finite $\cup$ Countably
Infinite

$|\text{ASCII}| = 128$, an alphabet

Great news: there is no
surjection:

$$f: \text{ASCII}^* \rightarrow \text{Power}(\text{ASCII}^*)$$

Hence take we'll explain

$f =$ LanguageRecognizedBy

so there exists a language that
is not recognized by any

$\left\{ \begin{array}{l} \text{Python} \\ \text{C++} \\ \text{APL} \\ \text{Fortran} \\ \text{Haskell} \end{array} \right\}$
 $\rightarrow$

program, and
we can describe such
a language "explicitly"

Admin

① You should send a test assignment to gradescope well before 11:59 pm on Thursday ...

② If you aren't yet on gradescope as of today:

email: jf@cs.ubc.ca

Subject: ... 421 ... (URGENT)

New today!

a LANGUAGE over Σ (an alphabet) is

- a subset of Σ^*
- an element of $\text{Power}(\Sigma^*)$

$(\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots \text{ strings over } \Sigma)$

e.g.

PRIMES = $\{2, 3, 5, 7, 11, \dots\}$

ODD = $\{1, 3, 5, 7, 9, 11, \dots\}$

over $\Sigma = \{0, 1, 2, \dots, 9\}$

PH.D-THESIS - UBC - U.P - 70-2015

language over ASCII
:
} finite
alphabet

ASCII^{*}

ASCII $\xrightarrow{\text{bijection}}$ $\{1, 2, \dots, 128\}$

ASCII^{*} $\xleftrightarrow{\text{bijection}}$ $\{1, 2, \dots, 128\}^*$

$\{1, 2, \dots, 128\}^*$

$= \{ \epsilon, 1, 2, \dots, 128,$

$(1, 1), (1, 2), \dots, (128, 128),$

$(1, 1, 1), (1, 1, 2), \dots \}$

A set, S , is countable infinite

if there is a bijection

$$f: S \rightarrow \mathbb{N} = \{1, 2, 3, \dots\}$$

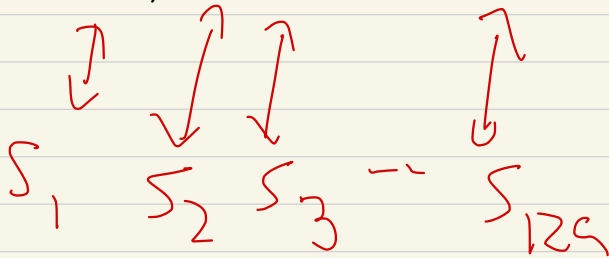
OR, equivalently

$$\dots \quad \mathbb{Z}_{\geq 0} = \{0, 1, 2, \dots\}$$

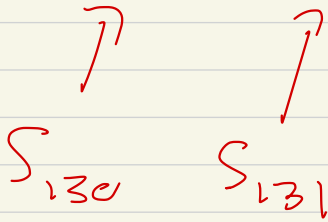
or $\mathbb{Q}$

$$\mathbb{Q} = \{\text{rational numbers}\}$$

$$11 \quad \{ \varepsilon, 1, 2, \dots, 128, \}$$



$$(1, 1), (1, 2), \dots, (128, 128),$$



$$(1, 1, 1), (1, 1, 2), \dots \quad \}$$

Equivalently, S is countably
infinite if

① S is infinite and $\exists$ surjection

$$f: \mathbb{N} \rightarrow S$$

② S is infinite and $\exists$ injection

$$f: S \rightarrow \mathbb{N}$$

$$(S, \mathbb{N} = \{1, 2, 3, \dots\})$$

$$\{1, 2\} \xrightarrow{\text{surjection}} \{1\}$$

$$1 \mapsto 1$$

$$2 \mapsto 1$$

Soon we'll see infinite sets

"larger" than $\mathbb{N}$, ASCII*,

such as $\mathbb{R}$, $\text{Power}(\mathbb{Q})$

uncountably
infinite

$\mathbb{C}$

$\uparrow$
reals

$$\mathbb{R} \xleftrightarrow{\text{bij}} \mathbb{C}$$

$$\mathbb{R} \subset \mathbb{C}$$

Scan ---

Thanks

M.P.W.

Power($\{0, 1, \dots, 9\}^*$)

injection
 $\longrightarrow \mathbb{R}$

Gr: 4.2 [5ip]

$\mathbb{R}$ are uncountably infinite

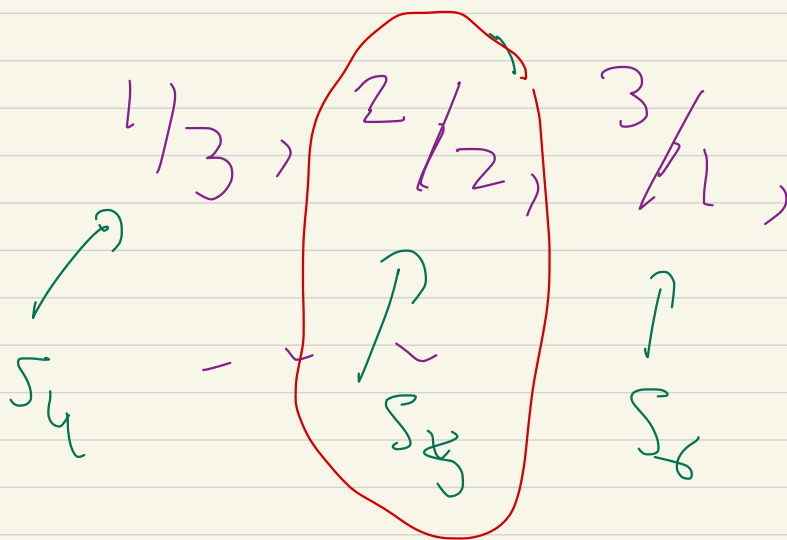
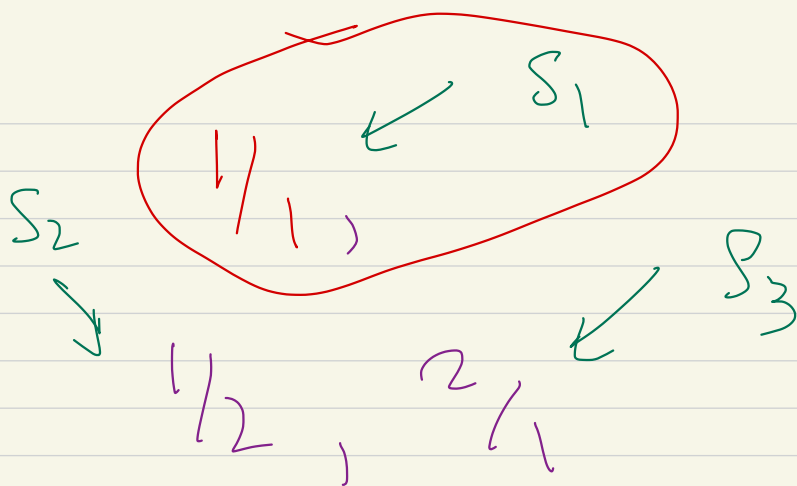
Power(ASCII^{*}) ~ 2ⁿ

Example: Let's show:

① $\mathbb{Q}_{>0}$ is countably infinite

	1	2	3	4	5	
1	$1/1$	$1/2$	$1/3$	$1/4$	$1/5$	...
2	$2/1$	$2/2$	$2/3$	$2/4$	$2/5$	...
3	$3/1$	$3/2$				
4	$4/1$					
5						

etc.



$S_1, S_2, S_3, S_4, S_5, S_6, \dots$

$S_i = i^{\text{th}}$ rational in seq
 $1/1, 1/2, 2/1, 3/1, 2/2, 1/3, \dots$

$$f: \mathbb{N} \rightarrow \mathbb{Q}_{>0} = \mathbb{Q}^+$$

$$i \mapsto S_i = i^{\text{th}} \text{ rational in sequence}$$

$$1/1, 2/1, 1/2, 3/1, \dots$$

s.t. f is surjective

$$\hookrightarrow \text{Size of } \mathbb{N} \geq \text{Size } \mathbb{Q}^+$$

$$|\mathbb{N}| \geq |\mathbb{Q}^+|$$

$1/1, 1/2, 2/1, 1/3, \cancel{2/2}, 3/1$

$4/1, 3/2, \dots$

Say that

① there is a surjection

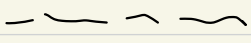
$$S \rightarrow T$$

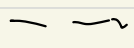
② there is a surjection

$$T \rightarrow S$$

Q: Is there a bijection $S \rightarrow T$

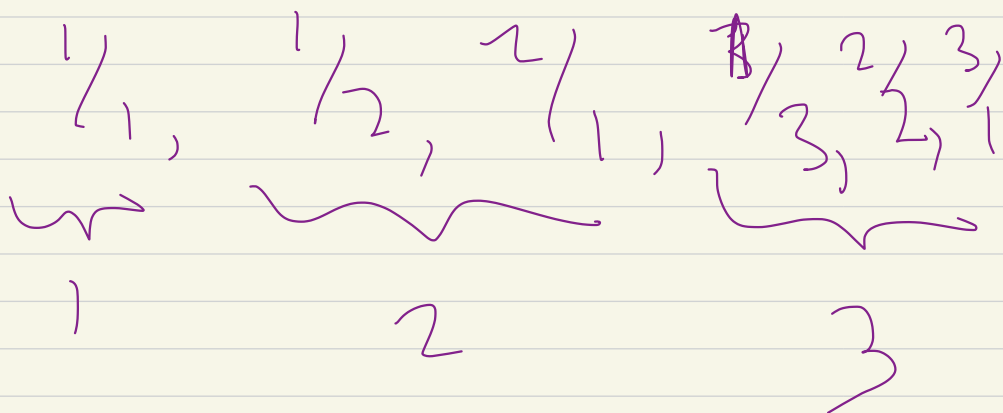
Homework due to J.F.

inspired by 

 A.H.



Class ends



Homework

$$SA_3 \rightarrow \binom{2}{2}$$

$$1 + 2$$

$$= 3 \quad \binom{3}{2}$$

$$1 + 2 + 3$$

$$= 6 \quad \binom{4}{2}$$

$$10 \quad \binom{5}{2}$$

$$15$$

"triangular numbers"

100th member

$$\binom{n}{2} \leq 100 \leq \binom{n+1}{2}$$

$$|S| < |T|$$

if $\nexists$ surjection $S \rightarrow T$

$\nexists$ injection $T \rightarrow S$

for us, they're the

same

There's no surjection