CPSC 421/501 Sept 8, 2025

- Diagonalization, Yes/No Tables

- $ASCII^*$, $\mathbb{N}$, $\mathbb{Q}$, $\mathbb{Z}$ All countably infinite

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- Countable vs. Uncountable

- P = people : $\{x, y, z\}$

2023

M = movies : $\{$Oppenheimer, Barbie, 2001, Encounters$\}$

$P \to \mathrm{Power}(M)$

"Injective form of Cantor's Theorem"

$f : P \hookrightarrow M$

$\text{HasSeen} : P \to \text{Power}(M)$

"Surjective form of Cantor's Theorem"

"Partial information form of Cantor's Theorem

$B = \{\text{Dispossessed}, \text{Lathe}, \text{Left}, \text{Leviathan}\}$

$S = \{\text{Ursula}, \text{Daniel}, \text{Ty}\}$

$\text{CoWrote} : S \to \text{Power}(B)$

LANGUAGES

Admin:

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Subject: __421___

On Wednesday if you haven't been added to Gradescope

HW #1 Group Homework

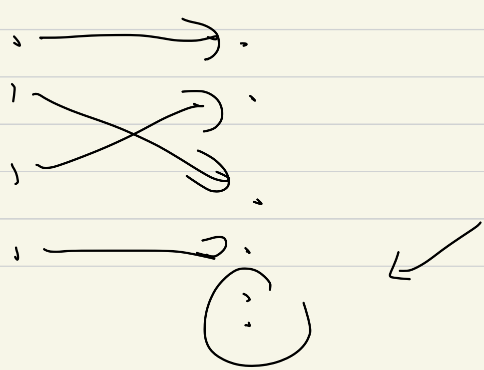
Thursday 11:59 pm

Cantor's Theorem: For any set S,

- S is smaller than $\text{Power}(S)$

(Cantor's Thm)':

For any set S, any map $f: S \to \text{Power}(S)$, f is not surjective



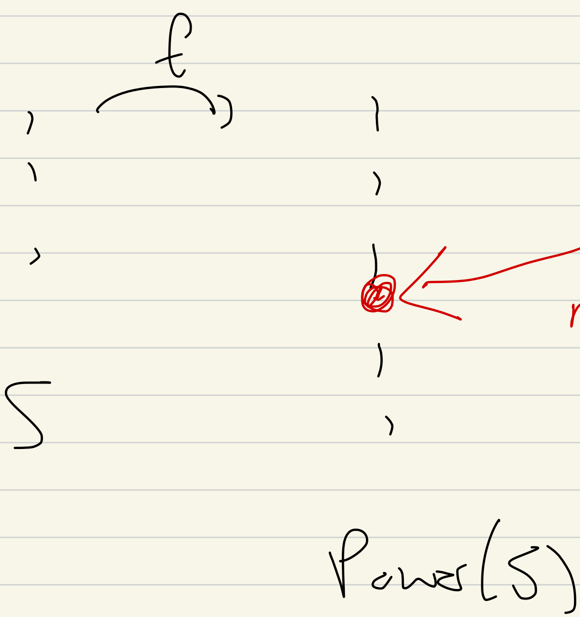
not in image of f

kinda like
- self-reference
- negation

Specifically:

$$T = \{ s \in S \mid s \notin f(s) \}$$

is not in the image of f.



not in the image of f

Countable vs. uncountable

ASCII = finite alphabet of 128 characters

a-z, A-Z, :, ;, ...

A set Σ is called an <u>alphabet</u> if it is finite and non-empty.

e.g. $\{a\}$, $\{a, b\}$, $\{1, 2, \dots, 2025\}$, ASCII

If Σ is an alphabet,

and $k = \mathbb{Z}_{\geq 0} = \{0, 1, 2, \ldots\}$

(rem: $\mathbb{N} = \{1, 2, 3, \ldots\}$

$\mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$

(Zahlen)

$$\Sigma^k = \underbrace{\Sigma \times \Sigma \times \ldots \times \Sigma}_{k \text{ copies}}$$

Chapter 0 of [Sip] Sipser's textbook

$$\Sigma = \{a, b\}$$

$$\Sigma^1 = \Sigma = \{a, b\}$$

$$\Sigma^2 = \Sigma \times \Sigma = \{a, b\} \times \{a, b\}$$

$$= \{(a,a), (a,b), (b,a), (b,b)\}$$

shorthand

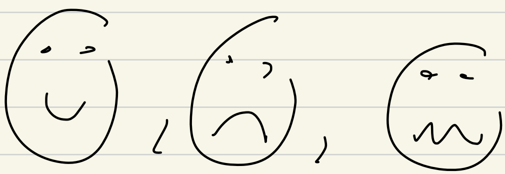
$$= \{aa, ab, ba, bb\}$$

Elements of Σ^k are called <u>strings</u> (words) <u>of length k over Σ</u>.

$\Sigma^3 =$ $\qquad$ ($\Sigma = \{a, b\}$)

$\{aaa, aab, aba, abb,$
$baa, bab, bba, bbb\}$

(convention that (if possible) we list words in lexicographical order)

(rem! $\{$  $\}$

$\Sigma^0 =$

$\Sigma = \{a, b\}$ $\quad |\Sigma| = 2$

$|\Sigma^3| = 8 = 2^3$

$|\Sigma^2| = 4 = 2^2$

$|\Sigma^1| = 2 = 2^1$

$|\Sigma^0| = 1$

$\Sigma^0 = \{\varepsilon\}$

ε = empty string

cartesian product

$$\Sigma \times \Sigma^2$$

$$\{a, b\} \{aa, ab, ba, bb\} = \{(a, aa), (a, ab), (a, ba), \ldots\}$$

concatenation

$$\{a, b\} \circ \{aa, ab\} = \{aaa, aab, baa, bab\}$$

$$\Sigma = \{0, 1\}$$

$$\Sigma^0 \circ \Sigma^2 = \Sigma^{0+2} = \Sigma^2$$

$$\{\varepsilon\} \circ \begin{Bmatrix} 00, \\ 01, \\ 10, \\ 11, \end{Bmatrix} = \begin{Bmatrix} \varepsilon \circ 00, \\ \varepsilon \circ 01, \\ \varepsilon \circ 10, \\ \varepsilon \circ 11 \end{Bmatrix}$$

$$= \begin{Bmatrix} 00, \\ 01, \\ 10, \\ 11 \end{Bmatrix}$$

The set of all strings over Σ,

$$\Sigma^* \stackrel{\text{def}}{=} \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots$$

$ASCII^* =$

$$\{\varepsilon\} \cup ASCII \cup ASCII^2 \cup ASCII^3 \cup \ldots$$

$$\{0,1\}^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}$$

convention:

list strings length 0

strings length 1

etc.

suggests the prime numbers

$$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots\}$$

Bad news! any

- Python program is an element of $ASCII^*$

- 30 page thesis $ASCII^*$

⋮

Claim

$$ASCII^* \xrightarrow{\text{bijection}} \underbrace{\{1, 2, 3, 4, \dots\}}_{\mathbb{N}}$$

$$\{a, b\}^* \xrightarrow{\text{bijection}} \{1, 2, 3, \ldots\}$$

$0 \mapsto a \qquad 1 \mapsto b$

$1\circ$ etc

ε		1	$\to$	1
a	0	10	$\to$	2
b	1	11	$\to$	3
aa	00	100	$\to$	4
ab	01	101	$\to$	5
ba	10	1		
bb	11	1		$\mathbb{N}$

$$\mathbb{Z}_{\geq 1} = \mathbb{N} = \{1, 2, 3, \ldots\}$$

$$\mathbb{Z}_{\geq 0} = \{0, 1, 2, 3, \ldots\}$$

$$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2 \ldots\}$$

$$\mathbb{N} \subsetneq \mathbb{Z}_{\geq 0}$$

$S \subset T$ S is a subset of T

(but $S = T$)

$S \subsetneq T$ —— (but $S \neq T$)

Is there a bijection

$$\mathbb{N} \to \mathbb{Z}_{\geq 0} \quad ?$$