CPSC 421/501 Sept 5, 2025

- Last time:

Ultimate goal:

(1) - How not to solve P vs. NP
- How to (maybe) solve P vs. NP

(2) First 2 weeks before the drop/withdrawl date:

- Uncomputability in CPSC 421

(3) $\{ S \mid S \text{ is a set s.t. } S \notin S \}$

(4) Cantor's Theorem

Admin:

(1) <u>Homework #1</u> (group work)

is due 11:59pm (Pacific Time)

September 11 (Thursday)

on gradescope, via Canvas (?)

[Homework #1 is "stand alone".]

(2) New problem this year!

(3) Current class sizes:

	421	501
Canvas	91	3
CS_UBC	102	3
Workday	92	3

Write to jf@cs.ubc.ca

Subject: -- 421 -- -- ?

Joel Friedman

Computer Science UBC

- Russell's Paradox is easy to gloss over. If the paradox is obvious, and obvious how to fix, then you might ask your favourite "AI" how long it took the planet's mathematical community to come to a standard way to resolve it.

The barber shaves each person
who doesn't shave themself

Cantor's Theorem: Let S be a set, and $f: S \longrightarrow \text{Power}(S)$ a function. Then f is not surjective; namely

$$T = \{ s \in S \mid s \notin f(s) \}$$

is not in the image of f.

[Namely, if you assume that $t \in S$ satisfies $f(t) = T$, then you get a contraction.]

Review!

Surjection

injection

bijection

image

Power(S)

$\in$, $\notin$, $\exists$, $\nexists$,

s.t. = such that

$\{ \text{blah} \mid \text{blahblahblah} \}$

Example of Cantor's Theorem:

$S = \{1, 2, 3\}$

$S = \{a, b, c\}$

$S = \{\text{workday}, \text{canvas}, \text{CS-list}\}$

Ultimately! (next week)

$S = \text{ASCII}^*$

$f: S \to \text{Power}(S)$

$f(s) \stackrel{def}{=} \{ i \in \text{ASCII}^* \mid \text{the program } s \text{ accepts } i \}$

Let $S = \{1, 2, 3\}$

$Pow(S) =$

$\{ \emptyset, \{1\}, \{2\}, \{3\},$

$\{1,2\}, \{2,3\}, \{1,3\},$

$S \}$

Let $f : S \to Pow(S)$

$f(1) = \{1, 2, 3\} = S$

$f(2) = \{ \} = \emptyset$

$f(3) = \{2\}$

$T = \{ s \in S \mid s \notin f(s) \}$

is $1 \in T$?

is $1 \notin f(1)$? $\quad 1 \in \{1, 2, 3\} = f(1)$

$1 \in f(1) \qquad 1 \notin T$

t \ is $t \in f(s)$	$f(1)$	$f(2)$	$f(3)$
1	Yes	?	?
2	?	No	?
3	?	?	No

is $t \in f(s)$

NO	YES	YES

$T = \{\ \ ,\ 2,\ 3\}$

Imaginary Student

$\geq 7'$ tall Yes

is not currently located in Vancouver

So, by analogy,

$T = \{2, 3\}$

cannot equal $f(1)$: $1 \in f(1)$

$1 \notin T$

$f(1) \neq T$

$f(2) = \emptyset, \qquad 2 \notin f(2)$

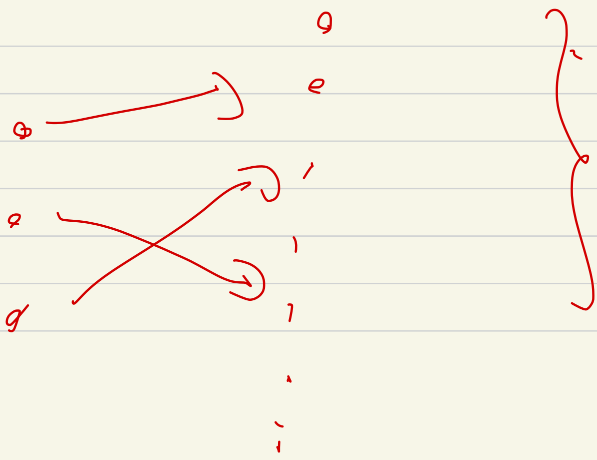
$2 \in T$

$f(2) \neq T$

$|S| = 3, \quad |Power(S)|$

$= 8 = 2^3$

$3 < 8$



no surjection

$$\left| \left\{ \text{Canvas 421 this morning} \right\} \right| = q_1$$

$$\left| \left\{ \text{workday} \right\} \right| = q_2$$

There is surjection

$$\underbrace{\left\{ \text{Canvas 421 this morning} \right\}}_{q_1} \longrightarrow \underbrace{\left\{ \text{workday} \right\}}_{q_2}$$

Note: $\forall\, n = 0, 1, 2, 3, \ldots \quad n < 2^n$

Let $f: S \to Pow(S)$,

let

$T = \{ s \in S \mid s \notin f(s) \}$

then $\nexists t \in S$

there exist no $t \in S$ s.t.

$f(t) = T$ for the

following reason:

(Proof by contradiction)

Either: $t \in f(t)$

or $t \notin f(t)$

If $t \in f(t)$:

$t \in T$

$t \notin T = f(t)$

If $t \notin f(t)$:

$t \notin T$, $T = \{ s \mid s \notin f(s) \}$

so false that $t \notin f(t)$

true " $t \in f(t)$