CPSC 421/501

Today:
- Admin
- Russell's Paradox
- Cantor's Theorem
  - ⇝ Some languages are
    - unrecognizable,
    - undecidable

§4.2 of [Sip]

Instructor: Joel Friedman

Course website:

https://www.cs.ubc.ca/~jf/courses/421.F2025/index.html

"In mathematics, you don't understand things. You just get used to them."

— John von Neumann

"In mathematics, it takes a while for things to 'sink in'."

— A. Friedman

Admin:

Grade:

$$0.55\, f + 0.35 \max(f, m)$$
$$+ 0.10 \max(f, m, h)$$

f = final, m = midterm,
h = homework (drop lowest 3).

Homework: Due on
Thursdays, 11:59 pm,
gradescope

Midterm: Friday, Oct 31, in class time.

Location: To be announced

Group Homework: Groups of size ≤ 4. One submission per group.

Individual Homework: One submission per student; must be your own work.

Some weeks we will not grade all homework parts.

[Homework must be legible.]

NOT OK! [illegible]

[illegible] QED

I will email via Workday today the piazza code

First 2 weeks!

1. Show the "halting problem" is "unsolvable," meaning ...

... "undecidable" (but it is "recognizable")

2. Give you an idea of the typical level of difficulty in CPSC 421/501

3. Start with "Uncomputability in CPSC 421/501"

Cantor's Theorem:

- We work in "naive set theory"
- Preliminary definitions:
  - Power(S), S a set
  - Image(f)
  - "f is surjective"

  f a function
- State Cantor's Theorem
- Give examples, formal proof, remarks

Russell's Paradox:

Let

$$T = \{ S \mid S \text{ is a set such that } S \notin S \}$$

Does $T \in T$?

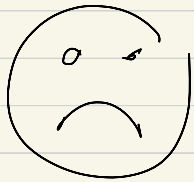
Or $T \notin T$?

$T = \{ S \mid S \text{ is a set}, \; S \notin S \}$

If $T \in T$ then

T is an S s.t. $S \notin S$

$\Rightarrow T \notin T$

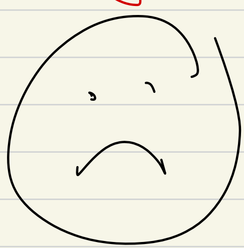


If $T \notin T$ then

T is not an S s.t. $S \notin S$

so $T \notin T$ is false

so $T \in T$



If S is finite, e.g.

$$S = \{a, b, c, \{a, b\}\}$$

Then

$$\{a, b, c, \{a, b\}\} \notin S$$

or otherwise

$$S = \{\_\_, \{a, b, c, \{a, b\} \ldots\}$$

So T above contains all finite sets

$$T = \Big\{ S \;\Big|\; \overset{S \text{ is a set}}{\underbrace{S \notin S}} \Big\}$$

- self-reference
- negation

→ Paradoxes

→ Sometimes you get a theorem

Power(S) = the set of all subsets of S.

<u>Cantor's</u> <u>Theorem</u>: If S is a set, and $f : S \to \text{Power}(S)$ is a function, then f is <u>not</u> <u>surjective</u>; i.e.

$S = \{1, 2, 3\}$

$\text{Power}(S) =$

$$\{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

$|S| = 3, \quad |\text{Power}(S)| = 2^3$

$f: \{1,2,3\}$ ← 3 elements

$\longrightarrow \text{Power}(\{1,2,3\})$ ← 8 elements

can't be surjective

There is some

$t \in \text{Power}(\{1,2,3\})$ s.t.

$\nexists\ s \in \{1,2,3\}$ s.t. $f(s) = t$