GROUP HOMEWORK 8, CPSC 421/501, FALL 2025

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Disclaimer: The material may sketchy and/or contain errors, which I will elaborate upon and/or correct in class. For those not in CPSC 421/501: use this material at your own risk...

Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(0) Who are your group members? Please print if writing by hand.

(1) Let $\Sigma = \{a\}$ and $\Sigma' = \{a, b\}$. Let L be a regular language over Σ, and let $M = (Q, \Sigma, \delta, q_0, F)$ be an m state DFA (i.e., $m = |Q|$) recognizing L. [1]
  (a) Give a **direct construction** of a DFA, $M' = (Q', \Sigma', \delta', q'_0, F')$ (therefore a DFA over Σ') that recognizes L (as language over Σ') with at most $m + 1$ states.
  (b) Use the **Myhill-Nerode theorem** to show that there is a DFA over Σ' that recognizes L (as language over Σ') with at most $m + 1$ states.

(2) Let $\Sigma = \{a\}$ and $\Sigma' = \{a, b\}$. Let L be a non-regular language over Σ.[2]
  (a) Use **the Myhill-Nerode theorem** to show that L is non-regular over Σ'.
  (b) Use an argument **other than the Myhill-Nerode theorem, but akin to one given in class**[3] to show that L is non-regular over Σ'. [Hint: we gave two types of arguments in class that would work.]

[1] This problem arose in discussions in 2025 with S.A.
[2] This problem arose in discussions in 2025 with S.A.
[3] e.g., don't use the pumping lemma.

(3) Say that $\Sigma \subset \Sigma'$ are two alphabets where Σ' is a strict superset of Σ (i.e., $\Sigma' \neq \Sigma$). Can your solutions to Problem (1a), (1b), (2a), and (2b) be easily modified to handle this more general situation? Explain **BRIEFLY (but supply enough details so that it is clear)** how to modify your solutions to (1a),(1b),(2a),(2b) (if you can) to this more general situation. [4]

(4) Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\rm acc}, q_{\rm rej})$ be a Turing machine such that for all $q \in Q$ and $\gamma \in \Gamma$, we have $\delta(q,\gamma) = (q', \gamma', \mathrm{R})$ for some $q' \in Q$ and $\gamma' \in \Gamma$, i.e., the tape head is only allowed to move right, never left. Describe a DFA $\tilde M = (\tilde Q, \Sigma, \tilde\delta, \tilde q_0, \tilde F)$ that recognizes the same language as does M; you must explicitly write down formulas for $\tilde Q, \tilde\delta, \tilde q_0, \tilde F$ and explain how your algorithm works. [5] [Hint (this may help...): if $\Gamma = \Sigma \cup \{\sqcup\}$, then this should seem very easy and intuitive. However, does the value of Γ really matter, given that you can only move right?]

(5) Bonus Question (worth 20% above the homework). Let $\Sigma = \{a\}$ and $\Sigma' = \{a, b\}$. Let L be a regular language over Σ, and let m and m' be, respectively, the smallest number of states needed in a DFA that recognizes L over, respectively, Σ and Σ'.
   (a) Characterize (i.e., describe, classify, etc.) the $L \subset \Sigma^*$ for which $m = m'$; **you should use the special properties of languages over $\{a\}$ learned in class to give as simple a characterization of L as possible.** [6] You can use the Myhill-Nerode theorem or some other method discussed in class.[7]
   (b) What if $\Sigma \subset \Sigma'$ and $\Sigma \neq \Sigma'$, but Σ can consist of more that one symbol. Characterize — in some way — the $L \subset \Sigma^*$ for which $m = m'$. Is your characterization as simple as the case $\Sigma = \{a\}$?

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[4]This problem arose in discussions in 2025 with S.A.
[5]We thank MdJ in 2025 for suggesting this problem.
[6]This problem inspired by discussions in 2025 with S.A.
[7]i.e., I don't recommend trying the pumping lemma here.