GROUP HOMEWORK 7, CPSC 421/501, FALL 2025

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(0) Who are your group members? Please print if writing by hand.

(1) Let $\Sigma = \{a, b\}$, $L_1 = \{a\}$, let L_2 be the language described by the regular expression $(a)^*$, i.e.,

$$L_2 = L_1^* = \{a^n \mid n \in \mathbb{Z}_{\geq 0}\} = \{a^n \mid n \in \mathbb{Z},\ n \geq 0\}.$$

(a) Use the Myhill-Nerode theorem to determine:
  (i) The minimum number of states in a DFA (over $\Sigma = \{a, b\}$) that recognizes L_1.
  (ii) The minimum number of states in a DFA (over $\Sigma = \{a, b\}$) that recognizes L_2.

(b) Write L_2 as recognized by the regular expression

$$(a)^*$$

Use the method in [Sip] 1.3 and briefly discussed at the of class on 10_10 to find a DFA recognizing L_2 by following the following steps:
  (i) Write a minimum state DFA, M, for the language $L_1 = \{a\}$.[1]

[1] We know that this is "unique up to isomorphism" by the Myhill-Nerode Theorem: Part 2, from the course handout "Non-regular Laguages and the Myhill-Nerode" and class discussion in 2025).

(ii) Viewing M as an NFA (as is, without omitting needless arrows or states because you are viewing M as an NFA), and write down an NFA, M', for $(L_1)^* = L_2$.

(iii) Use the procedure in [Sip], Section 1.3 to convert the NFA M' to a DFA, M'' that recognizes L_2 [Hint: You likely are starting with an 8 state DFA, but you can omit needless states.]

(2) Let $\Sigma = \{a, b\}$. Let L be the language described by regular expression $a^*b^*b^*a^*$.
  (a) Write down a DFA that recognizes L, any way you like.
  (b) Use the Myhill-Nerode theorem to determine the minimum number of states needed in a DFA that recognizes L.

(3) Let $\Sigma = \{a, b\}$. Let $k \in \mathbb{N}$, and let L be the language described by regular expression $\Sigma^* a \Sigma^{k-1}$, hence

$$L = \{s \in \{a, b\}^* \mid \text{the } k\text{-th to last symbol of } s \text{ is an "}a\text{"}\}.$$

Use the Myhill-Nerode theorem to determine the minimum number of states needed in a DFA that recognizes L.

(4) Bonus Question (worth 20% above the homework). Say that a lanugage, L, is recognized by an NFA with q states, over an alphabet Σ. Show that there an algorithm that given a string, $s \in \Sigma^n$, determines whether or not $s \in L$, in time $O(q^2|\Sigma|n)$, where "$O(\ )$" and "time" is in the sense of CPSC 320.[2][3] If you can't quite do this, find a time that $O(p_1(q)p_2(|\Sigma|)p_3(n))$, where p_1, p_2, p_3 are polynomials (preferably of as low degree as possible).

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[2] Thanks to Auden, 2025, for the correction.

[3] If you haven't taken CPSC 320, this should be in the sense of how many "steps" a Python algorithm/program would take, assuming all variables are of a **fixed** number of bits (Python allows for variables of arbitrary length, which would allow for a faster algorithm).