GROUP HOMEWORK 3, CPSC 421/501, FALL 2025

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Disclaimer: The material may sketchy and/or contain errors, which I will elaborate upon and/or correct in class. For those not in CPSC 421/501: use this material at your own risk...

Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(0) Who are your group members? Please print if writing by hand.

(1) What is wrong with the following proof that there exists no surjection from $\mathbb{N} \to \mathbb{Q} \cap [0,1)$ (the set of rational numbers, r, satisfying $0 \le r < 1$):[1] *Assume there is a surjection $\mathbb{N} \to \mathbb{Q} \cap [0,1)$, i.e., there is a sequence $r_1, r_2, r_3, \ldots$ such that each element of $\mathbb{Q} \cap [0,1)$ appears (at least) once. Write out the decimal expansion of each r_i (as we did when we proved that $\mathbb{R} \cap [0,1)$ is uncountable). Let r be the number whose i-th digit (i.e., the digit in the 10^{-i} place) equals*

$$\begin{cases} 1 & \textit{if } r_i\textit{'s } i\textit{-th digit is not a } 1\textit{, and} \\ 2 & \textit{otherwise.} \end{cases}$$

Hence r is not equal to any of the r_i. Hence, by contradiction, we see that there is no surjection from $\mathbb{N} \to \mathbb{Q} \cap [0,1)$.
(I've put all of this in *italics* to express the fact that there is an error somewhere.)

(2) Exercise 9.2.6, parts (a)–(c), on the handout "Uncomputability in CPSC421/501."

[1] Thanks to ARS for suggesting this type of question.

(3) Let $f\colon A \to B$ and $g\colon B \to C$ be functions on sets A, B, C. Say that gf is injective and, in addition, $|A| = |B|$ (meaning there is a bijection from A to B), but A, B can be infinite sets. Is g necessarily injective? (Give a proof that g is injective or give examples of f, g where g is not injective).

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