CPSC 421/501 Dec 6, 2023

- Final 2021, Part 1, Q 1, 5th T/F

Is there a surjection $\mathbb{N} \to \{a,b\}^*$

- Office Hours before the final

Mine: ICCS 104

Th 2-3, Fri 11-12:30

Mon 11-12:30

Dec 11

$\exists$ surjection $\mathbb{N} \to \{a,b\}^*$

$$\Sigma^*_{ASCII} = \{ i_1, i_2, \dots \}$$

class notes: we did this for

$\{a,b\}^*$ instead!

$$= \begin{array}{l} \{ \varepsilon, \; a, \; b, \; aa, \; ab, \; ba, \; aa, \dots \} \\ \;\;\uparrow \;\;\; \uparrow \;\;\; \uparrow \\ \;\; i_1 \;\; i_2 \;\; i_3 \end{array}$$

bijection $\mathbb{N} \to \{a,b\}^*$

$\mathbb{N} \to \Sigma^*$

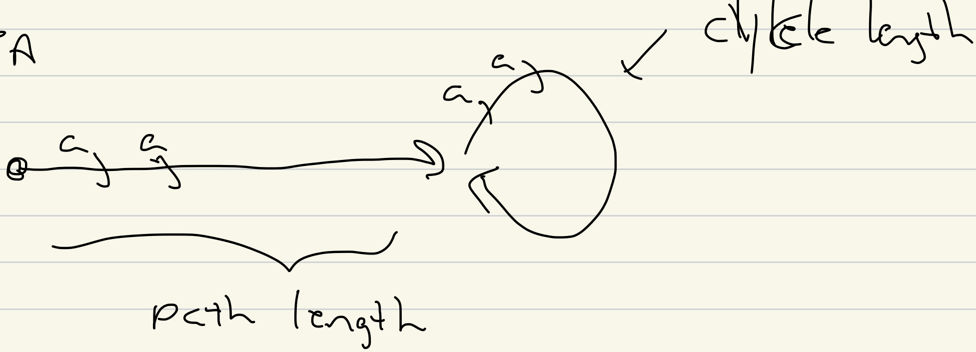
Not on final! countable sets.

S Countable: $\exists$ surjection $\mathbb{N} \to S$

New this year 2023, over 2021

$\Sigma = \{a\}$

DFA



Shortest DFA: find period of L

(1) the smallest p s.t. for some n_0,

for $n \geq n_0$: $x^n \in L \iff x^{n+p} \in L$

(2) If p period of L, n_0 smallest possible, smallest DFA for L has $n_0 + p$ states

e.g.

$$L = \{ a^{12}, a^{19}, a^{26}, \ldots \}$$

$$= \{ a^n \mid n \bmod 7 = 5, \text{ and } n \geq 12 \}.$$

$\text{period}(L) = 7$

smallest $n_0 = 6$???

could n_0 be 5 ?? No

$a^5 \notin L$, but $a^{12} \in L$

but for $n_0 = 6$, membership in L

depends on $n \bmod 7$:

a^6, $a^{6+7} = a^{13} \notin L$

a^7, $a^{14} \notin L$

a^8, a^{15}

$\vdots$

etc.

$a^{12} \in L$, $a^{12+7} \in L$ but

$a^{12-7} \notin L$

In 2023, we spent time on

(circuit complexity) and

formula complexity

$Th_{2,n}$

$Parity_n$

Subbotovskaya's

$$Parity_n \geq n^{3/2}$$

"method of restrictions"