CPSC 421/501 Nov 17

Thm (Cook-Levin): $SAT \in P$

$\Rightarrow P = NP$.

Similarly: $3SAT \in P \Rightarrow P = NP$.

Similarly for 3SAT replaced with 3COLOR, SUBSET-SUM, PARTITION, etc.

"NP-complete"

Midterm:

Avg 28.15 / 35

Median 31/35

Rem:

(1) 5/5 for a problem

may still have corrections

(2) Keep doing the

homework ...

Homework	9	due	Nov 23
"	10	"	Nov 30
"	11	"due"	Dec 7
			(but not collected)

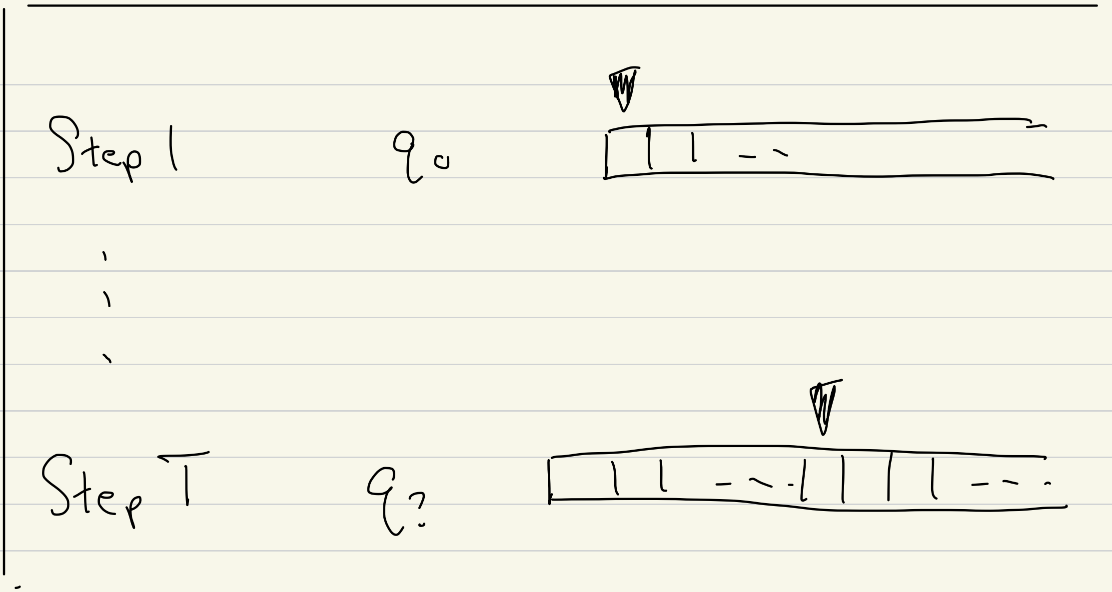
Last class Dec 6 (Wed)

Classes end Dec 7 (Thu)

Exams: Dec 11–22

Our exam: Dec 11, 3:30pm

Extra office hours: Dec 4-8



Fix a non-det T.m., M:

$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$ is a non-deteministic T.m.

Input = $\sigma_1 \ldots \sigma_n$

c, k fixed

① Here $T = cn^k$, n = length of input

② Tape head position at step t is cell # $\leq t$

③ $X_{ijr} = \begin{cases} T & \text{if cell}[i,j] \text{ has } r \\ F & \text{otherwise} \end{cases}$

$Y_{ij} = \begin{cases} T & \text{if tape head on cell}[i,j] \\ F & \text{otherwise} \end{cases}$

$Z_{iq} = \begin{cases} T & \text{if in state } q \text{ at step } i \\ F & \text{otherwise} \end{cases}$

$i, j \in \{1, \ldots, T\}$, $r \in \Gamma$, $q \in Q$

Boolean formula

$f = f(u_1, \ldots, u_N)$ is a

3-Conjunctive Normal Form

formula if

$$f = (w_{11} \text{ OR } w_{12} \text{ OR } w_{13})$$
$$\text{AND } (w_{21} \text{ OR } w_{22} \text{ OR } w_{23})$$
$$\text{AND } (\qquad\qquad)$$
$$\vdots$$
$$\text{AND } (w_{\ell 1} \text{ OR } w_{\ell 2} \text{ OR } w_{\ell 3})$$

$$w_{ij} = u_1, \ldots, u_N, \neg u_1, \neg u_2, \ldots \neg u_N$$

$$\text{SAT} = \left\{ \langle f \rangle \;\middle|\; \begin{array}{l} f \text{ Boolean formula} \\ \text{that is satisfiable} \end{array} \right\}$$

$$\text{3SAT} = \left\{ \langle f \rangle \;\middle|\; \begin{array}{l} f \text{ Boolean formula} \\ \text{that is satisfiable} \\ \text{and } f \text{ is in} \\ \text{3CNF} \end{array} \right\}$$

have this in mind

$$X_{ijr}, \; Y_{ij}, \; Z_{iq} \ldots$$

So far: the T.m has a computation path that accepts input $\sigma_1 \dots \sigma_n$ iff

(1) Config at step 1 represents what the initial T.m. config should be

AND

There is a way to move step 1 to step 2

AND

" " " " " " " step 2 to step 3

⋮

" " " " " " " to step T

$(T = cn^k)$

AND

the state in step $T = Cn^k$

is q_{accept}

So ...

$$x_{ij\gamma} = \begin{cases} T & \text{iff cell}[i,j] \\ & \text{has a } \gamma \end{cases}$$

$i, j = 1, \ldots, T, \qquad T = Cn^k$

$\gamma \in \Gamma$ tape alphabet

Condition: cell$[i,j]$ has exactly

one symbol from Γ

$i=1,\ j=1\ ,\quad \Gamma = \{\gamma_1, \ldots, \gamma_{N_\Gamma}\}$

$$\leftarrow \boxed{(X_{11\gamma_1} \text{ or } X_{11\gamma_2} \text{ or } \ldots \text{ or } X_{11\gamma_{N_\Gamma}})}$$

AND

$$(\neg X_{11\gamma_1} \text{ or } \neg X_{11\gamma_2})$$

AND

$$(\neg X_{11\gamma_1} \text{ or } \neg X_{11\gamma_3})$$

AND

$$\vdots$$

$$(\neg X_{11r_1} \text{ or } \neg X_{11r_2} \text{ or } \neg X_{11r_2})$$

, —— - -

Problem: $u_1, \dots, u_{N_r}$

Express

$(u_1 \text{ or } u_2 \text{ or } \dots u_{N_r})$

as 3 CNF (Homework)

Namely, for example

$$u_1 \text{ or } u_2 \text{ or } u_3 \ldots \text{ or } u_6 = T$$

iff

$$\left.\begin{array}{l} (u_1 \text{ or } u_2 \text{ or } w_1) \text{ AND} \\ (\neg w_1 \text{ or } u_3 \text{ or } w_2) \\ \qquad\qquad \text{AND} \\ (\neg w_2 \text{ or } u_4 \text{ or } w_3) \\ \qquad\qquad \text{AND} \\ (\neg w_3 \text{ or } u_5 \text{ or } u_6) \end{array}\right\} \text{3 CNF is satifiable}$$

We add new variables $w_1, \ldots, w_3$

Similarly :

for each i, we want exactly one of

$Y_{i1}, Y_{i2}, \ldots, Y_{iT}$ to be true, here $T = cn^k$

$Y_{ij} = T \iff$ tape head at step i is on cell pos j

there are $T = cn^k$ variables

Also, exactly one of

$Z_{iq_1}, Z_{iq_2}, \ldots, Z_{iq_{N_Q}}$ is true

Aside: say non-det T.m., M, and an input length n, you produce a 3CNF formula of size $10^6 n^{17}$ s.t. the formula $\in$ 3SAT iff the T.m. can accept the input in time $4n^2$.

Say that you have a SAT solver that for a formula of length N, solves SAT in time $3000 N^{12}$

So: non-det TM accepts input length n it time $4n^2$

iff some 3CNF is satisfiable,

3CNF length $10^6 n^{17} = N$,

so the SAT solver takes time

$$3000 N^{12}$$

$$= 3000 \left(10^6 n^{17}\right)^{12}$$

$$= O\left(n^{17 \cdot 12}\right)$$

Using

polynomial (another polynomial)

= some other polynomial

AND (   )

(   )

AND (   )

⋮

AND $(z_{1q_{12}}$ OR $z_{1q_{12}}$ OR $z_{1q_{12}})$

initial state q_{12}