CPSC 421/501

- Today:

§4.2 [Sip]

§1-6 "Uncomputability in CPSC 421/501"

For all class today:

$$\Sigma_{TM} = \{0, 1, \ldots, 9, \#, L, R\}$$

$$\parallel$$

Σ for today

For each standard TM, M

$\langle M \rangle$ = description of M with Σ_{TM} symbols

Also view:

$$\begin{array}{lcccccccc} \Sigma_{TM} = \{ & 0, & 1, & \_, & q, & \#, & L, & R & \} \\ \text{identity with} & \uparrow & \uparrow & & \uparrow & \uparrow & \uparrow & \uparrow & \\ \{ & 1, & 2, & \ldots, & 10, & 11, & 12, & 13 & \} \end{array}$$

$$= \Sigma_{\text{standard TM}}$$

So $\left\{ \begin{array}{l} \text{decidable} \\ \text{recognizable} \end{array} \right\}$ languages

Let M be a T.M. with input symbols Σ.

For any $i \in \Sigma^*$,

$$\text{Result}(M, i) = \begin{cases} \text{yes}, & \text{if } M \text{ accepts } i \\ \text{no}, & \text{if } M \text{ rejects } i \\ \text{loops}, & \text{otherwise, i.e., if } M \text{ doesn't halt on input } i \end{cases}$$

=

For each T.M., M, on input symbols, Σ, we let

$$\text{LanguageRecBy}(M) = \{ i \in \Sigma^* \mid \text{Result}(M, i) = \text{yes} \}$$

A T.M., M is a <u>decider</u> if

$\forall i \in \Sigma^*$

Result(M, i) $\neq$ loops

i.e. = yes, no

We say $L \subset \Sigma^*$ is <u>recognizable</u>

if for some T.M., $M = (Q, \Sigma, \ldots)$

L = LanguageRecBy(M)

We say that $L \subset \Sigma^*$ is decidable

if there is a decider, M,

s.t.

$L = \text{LanguageRecBy}(m)$.

Example:

ACCEPTANCE_{TM}

$$= \{ \langle m, i \rangle \mid \text{Result}(m,i) = \text{yes} \}$$

is recognized by a universal T.M. (with some conventions at the end).

Theorem: ACCEPTANCE_{TM}

$\subset \{0, \ldots, 9, \#, L, R\}^*$ is not decidable.

First: define negation, $\neg$,

$\neg$ yes = no, $\neg$ no = yes,

$\neg$ loops = loops.

=

Lemma: Say that H recognizes ACCEPTANCE_{TM}. Let D be a T.M., s.t. for any standard TM, M,

$$\text{Result}(D, \langle M \rangle)$$

$$= \neg\ \text{Result}(H, \langle \langle M \rangle, \langle M \rangle \rangle)$$

$$= \neg\ \text{Result}(H, \langle M, \langle M \rangle \rangle)$$

(here $\langle M \rangle$ is description of M

$\langle M, i \rangle$ " " M, i

$\langle M, \langle M \rangle \rangle$ " " $M, \langle M \rangle$)

Rem: For T.M. (Python progs, ...)

given H you can build D.

Then:

1. $\text{Result}(D, \langle D \rangle) = \text{loops}$

(i.e. $\neq$ yes, no)

2. $\text{Result}(H, \langle D, \langle D \rangle \rangle) = \text{loops}$

Hence H, D are not deciders.

$=$

Pf: Say $\text{Result}(D, \langle D \rangle) = \text{yes}$

$\vdots$

$\text{Result}(D, \langle D \rangle) \neq \text{yes}$

Details

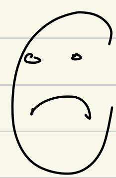
$\text{Result}(D, \langle D \rangle) = \text{yes}$

$\text{Result}(H, \langle D, \langle D \rangle \rangle) = \text{no}$

rec ACCEPTANCE

D does not accept $\langle D \rangle$

$\text{Result}(D, \langle D \rangle) \neq \text{yes}$



Similarly if $\text{Result}(D, \langle D \rangle) = \text{no}$

Cor:

$\Sigma^* \setminus$ ACCEPTANCE

is not recognizable:

since

ACCEPTANCE is recognizable

and

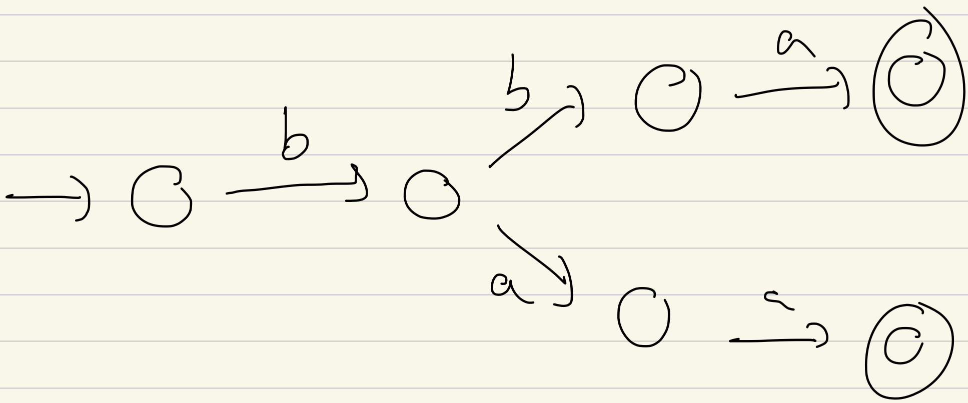
$\Sigma^* \setminus$ ACCEPTANCE is "

$\Rightarrow$ decidable

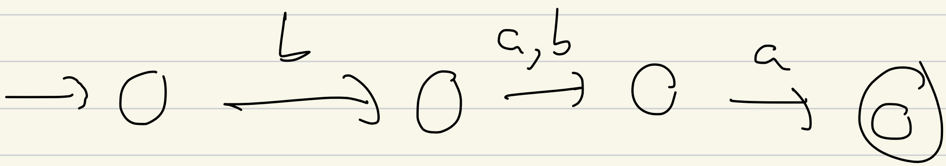
which is impossible

$L, \Sigma^* \setminus L$ are recog $\Rightarrow$ $L, \Sigma^* \setminus L$ decidable

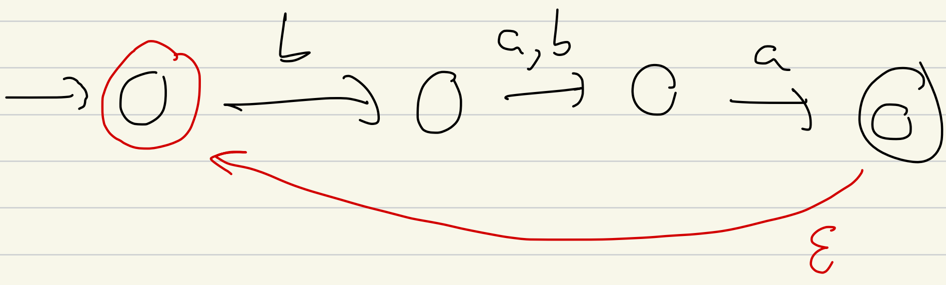
$\{bba, baa\}$



OR



$\{bba, baa\}^*$



$\{a, b, c\}$ Quest 4

2019 Midterm

$L = \{ s \in \{a, b, c\} \mid$ exactly half of symbols $= c \}$

$\text{Accept}_L(\varepsilon) = \{c^0, \ldots\}$

" $(a) = \{c, \overset{cacc b}{\ldots}\}$

" $(a^2) = \{c^2, \ldots\}$

" $(a^3) = \{c^3, \ldots\}$

You can quote results

on HW