CPSC 421/501 Oct 30, 2023

- Partial solutions to midterm practice now on exam webpage

- Today:
  - Universal TM
  - Recognizing ACCEPTANCE
  - Etc.

Universal TM:

TM: 7-tuple:

"standardize"

$$\begin{array}{ccccccc} & & & & 1 & 2 & 3 \\ (Q, & \Sigma, & \Gamma, & \delta, & q_0, & q_{accept} & q_{reject}) \end{array}$$

$\{1, \ldots, n_\Gamma\}$

$\{1, \ldots, n_Q\}$ $\{1, \ldots, n_\Sigma\}$

Described as string over

$\{0, \ldots, 9, \#, L, R\}$

$$\delta: Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$$

So:

$\langle p, i \rangle$

input

- was a Python program
- now a Turing machine

For TM:

p

$30\#30\#60\#\ \underbrace{\delta(1,1)}_{5\#3\#R}\ \#\ \ldots\ \#\ \delta(n_Q, n_\Gamma)$

\# input

[input: 2#1#3#3#2]

Universal Turing machine, takes

$\langle p, i \rangle$ (or $\langle M, i \rangle$)

simulate

tape 1 input: $\langle M, i \rangle$

tape 2: copy M: $|3|0|\#|3| \ldots \overbrace{- - \; (\#|5|\#|1|\#|R}^{\delta}$

tape 3: copy i: 02#01#03#12#30#27

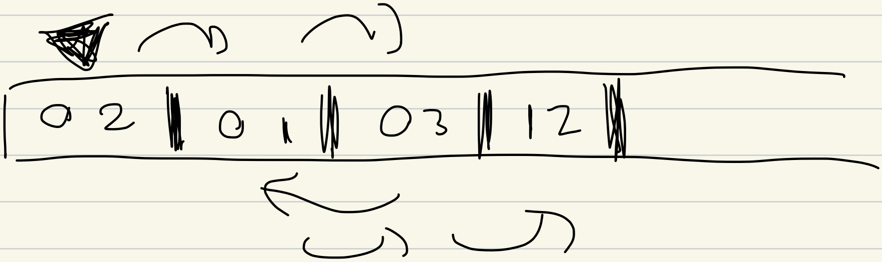
tape 4

tape 5

tape 6

idea

Intuition



State

$$\{01, 02, \ldots, 30\}$$

q_0 $\quad$ q_{acc} $\quad$ q_{rej}

$$\Sigma = \{01, 02, \ldots, 30\}$$

$$\Gamma = \{01, 02, \ldots, 30, 31, \ldots 60\}$$

blank

tape 3: 02 # 01 # 03 # 12 # 30 # ...

keep tape head over the simulated computation tape

sometime: simulated situation

55	09	03	12	30	27	⊔	⊔
						31	31

tape 4: we see

0	3

tape 5: what state

1	5

tape 6:

3	0

# of states

tape 7: $\boxed{6|0|}$ # tape symbols

tape 8: write $\delta(1,1), \ldots, \delta(n_Q, n_\Sigma)$

$\delta(1,1)$

12 # 7 # R #

⋮

This multi-tape TM can simulate M on input i.

We can therefore: running, U, our universal TM, we can

accept $\langle M, i \rangle$ if M accepts i

reject $\langle M, i \rangle$ " " rejects i

we never stop " " loops i

U doesn't halt " " loops i.

Hence U accept $\langle M, i \rangle$ iff

iff M accepts i, so

LanguageRecBy(U) =

$\{ \langle M, i \rangle \mid M \text{ accepts } i \}$

$= \text{ACCEPTANCE}_{TM}$

U recognizes ACCEPTANCE_{TM}

In [Sip] §4.2

Now: ACCEPTANCE_{TM} is

undecidable.

Sample Midterm Questions

$$\left\{ (ab)^{2n+1} \;\middle|\; n = \mathbb{Z}_{\geq 0} = \{0, 1, 2, 3, \ldots\} \right\}$$

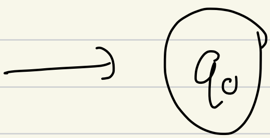
$$= \{ ab, \underset{(ab)^3}{ababab}, (ab)^5, \ldots \}$$

$$= L$$

Find the minimum # of states

$$L \leftrightarrow (ab)(abab)^*$$

Myhill-Nerode



q_0 initial state

where ε is taken to

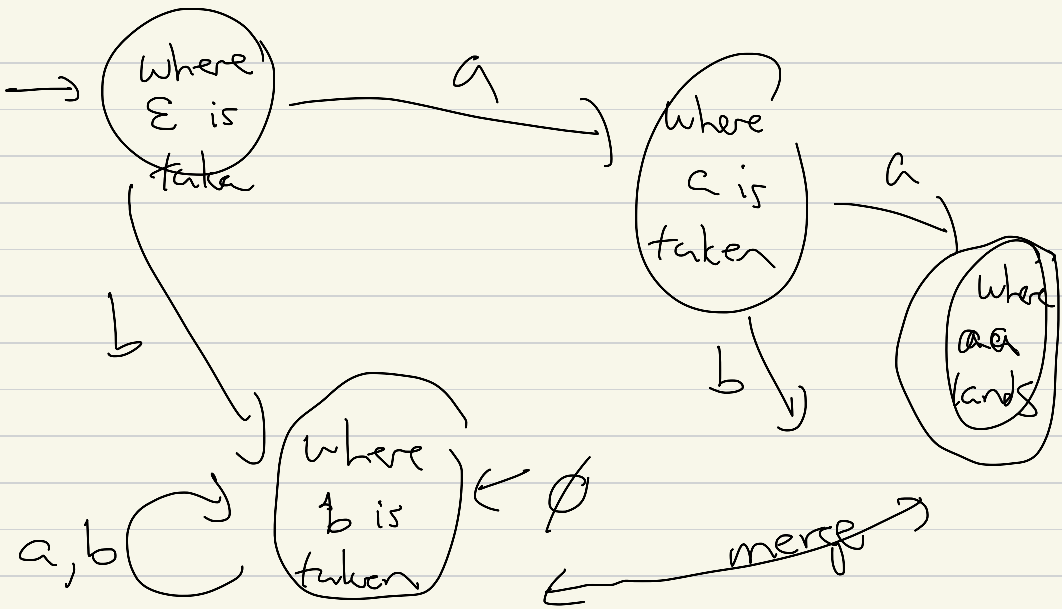
corresponds to

all $w \in \{a,b\}^*$

s.t.

$$\mathrm{AccFut}_L(w) = \mathrm{AccFut}_L(\varepsilon) = L$$

$$= (ab)(abab)^*$$



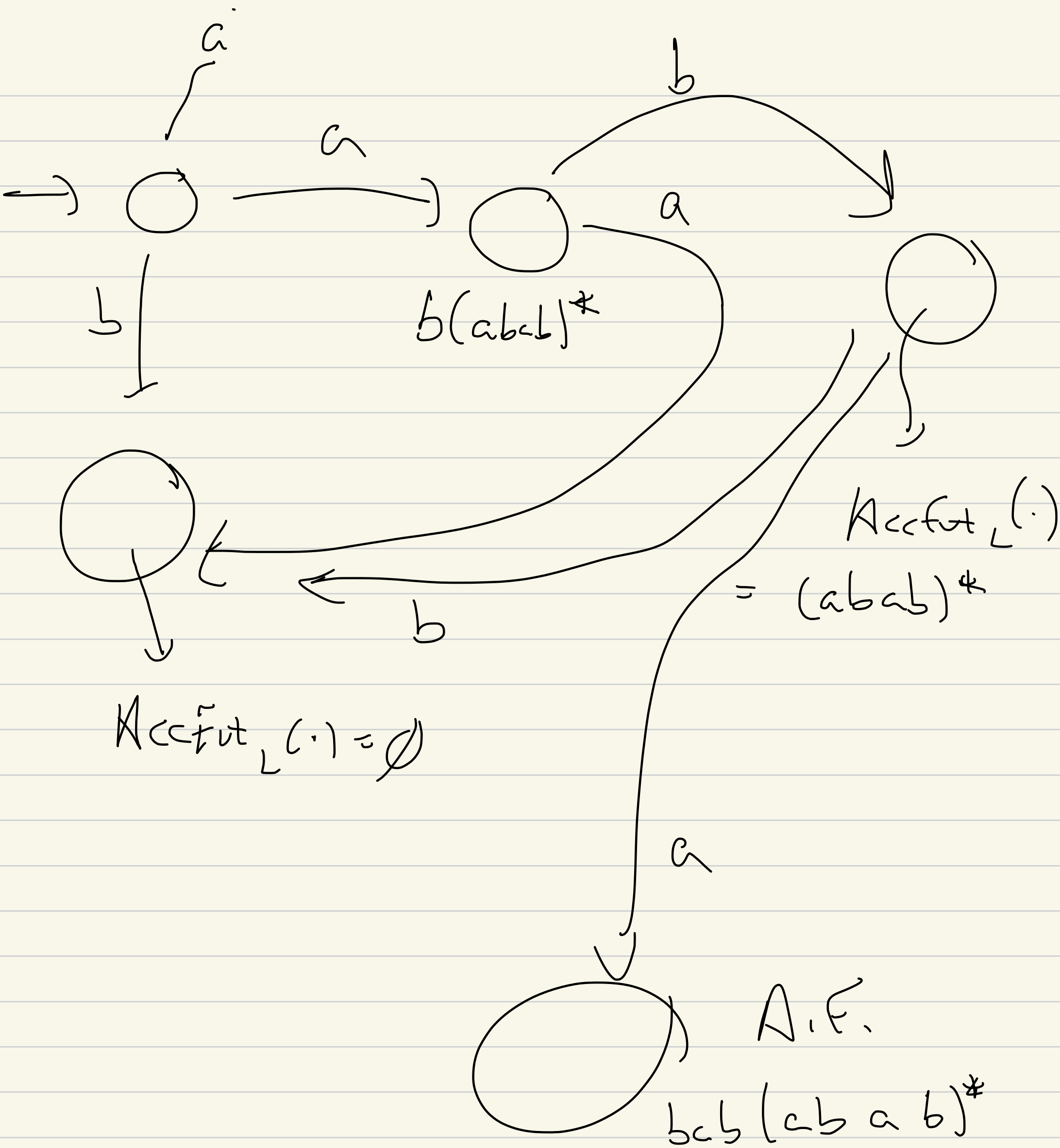
$$\mathrm{AccFut}_L(a) \doteq \{ s \mid as \in L \}$$

$$= b\,(abab)^*$$

$$\mathrm{AccFut}_L(b) = \{b\} \circ \{abab\}^*$$

$$= \{ s \mid bs \in L \} = \emptyset$$

$$\mathrm{AccFut}_L(aa) = \emptyset$$



$= bL$

$$AF_L(aba) = \{ abas \in L \} = b^{ab}(abab)^*$$