CPSC 421/551

Midterm: Friday, Nov 3.

Location: In class

- You may use 1 two-sided sheet of notes
- Exam will be 45 minutes long
- You will be seated in alphabetical order; please remain outside until you are asked to come in.
- Exam study guide now available online
Monday, Oct 30 —
20 minutes for midterm practice questions

Wednesday, Nov 1 —
30 minutes for midterm practice questions

2023:Same as 2021:
(1) 2023: Decidability & Recognizability
not countable sets (2021)
(2) 2023 some different HW problems
Today: Build a universal Turing machine.

Motivation: to recognize the acceptance problem:

**ACCEPTANCE** Turing machines

\[ \text{def} = \{ \langle M, i \rangle \mid \begin{align*} & M \text{ is a } \text{TM}, \\ & i \text{ is an input} \\ & \text{to } M \text{ s.t.} \\ & M \text{ accepts } i \end{align*} \} \]

Description of a TM, \( M \) and an input, \( i \), to \( M \) ...
Formally, a TM is

\((Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\),

where 

- \(Q\) is any finite set

Too many finite sets...

🤔 Standardize TM's:

A standard TM is

\((Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\),

such that:
\[ Q = \{1, 2, 3, \ldots, \, n_Q\} \]

where \( n_Q \in \mathbb{N} = \{1, 2, \ldots\} \)

\[ \Sigma = \{1, 2, 3, \ldots, \, n_{\Sigma}\} \]

\[ \Gamma = \{1, 2, \ldots, \, n_{\Sigma}, \, n_{\Sigma} + 1, \ldots\} \]

where \( n_{\Sigma} \in \mathbb{N} = \{1, 2, \ldots\} \)

where \( n_{\Sigma} \in \mathbb{N} \) s.t. \( n_{\Sigma} \geq n_{\Sigma} + 1 \)

And: \( q_0 = 1, \, q_{\text{accept}} = 2, \, q_{\text{reject}} = 3 \)

\( (n_Q \geq 3) \), blank symbol = \( n_{\Sigma} + 1 \)
Describe this TM: we can give $N_Q, N_{\Sigma}, N_r$ and description of $\delta$ 

E.g. $n_Q=20, n_{\Sigma}=5, n_r=10$

20 # 5 # 10 # describe $\delta$

(So far we use symbols 0,1,2,...,9, #)
\[ \delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\} \]

for 1-tape TM

\[ \forall q \in \{1, \ldots, n\} \]

\[ \gamma \in \{1, \ldots, n\} \]

we need to describe \( \delta(q, \gamma) \)

\[ \delta : \delta(1,1), \delta(1,2), \ldots, \delta(1,n) \]

\[ \delta(2,1) \quad \delta(2,2) \quad \ldots \]

\[ \vdots \]

\[ \delta(n,1) \quad \ldots \quad \delta(n,1) \]
Maybe!

\[ \delta(1, 1) \# \quad \delta(1, 2) \# \quad \cdot \quad \cdot \]

\[ \# \delta(n_q, n_r) \]

some some tape
integer symbol L or R

in in

\{1, \ldots, 9\}, \{1, \ldots, 3\}

e.g.

3 \# 2 \# L \# \#

R

Alphabet for TM: \{0, \ldots, 9, L, R, \#\}
If $\mathbf{m}$ is standard TM looks like
\[
\begin{array}{cccccc}
2 & 0 & \# & 6 & \# & 1 & 0 & \# & 3 & \# & 9 & \# & 7 & \#
\end{array}
\]
\[
\delta(1, 2) \rightarrow \begin{array}{c}
1 & 9 & \# & 3 & \# & R & \#
\end{array}
\]

\[<\mathbf{m}, \mathbf{i}> = <\mathbf{m}> \text{ then } <\mathbf{i}> \]

\[i \in \Sigma^* = \{1, \ldots, n\}^* \]

e.g. $\Sigma = \{1, \ldots, 6\}$

5 # 4 # 1 # 1 # 3
Say \( \Sigma = \{ \cdot, \ldots, N \} \) and you have a TM that recognizes \( L \subseteq \Sigma^* \).

Let \( Tm = (Q, \Sigma, \Gamma, \ldots) \) be a standard TM.

Is there a standard TM that recognizes \( L \)?

Say \( Q = \{ q_0, q_{acc}, q_{rej}, q_5, q_{mod3} \} \):

\[
\begin{array}{cccccc}
& 1 & 2 & 3 & 5 & 4 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
1 & 2 & 3 & 5 & 4
\end{array}
\]
Take
\[ \mathcal{C} \subseteq \Sigma = \{\#\} \cup \{\alpha, \beta, \gamma\} \]
\[ \downarrow \downarrow \downarrow \downarrow \]
\[ \{1, \ldots, n\} \cup \{n+1, n+2, n+3, n+4\} \]

= 

Again!

Say TM for \( \Sigma = \{\alpha, \beta, \gamma\} \)
\[ \downarrow \downarrow \downarrow \downarrow \]

no harm! \( \Sigma = \{1, 2, 3\} \)
Now... build a universal TM:

\( \langle M \rangle \) followed by \( \langle i \rangle \)

Standardized

\[ 2G \# 5 \# 12 \# 3 \# 2 \# L \]

\[ \# 4 \# 7 \# R \]

\[ \# \]

\[ \# 9 \# 1 \# L \]

\( \langle m \rangle \) → \( \# 3 \# 3 \# 2 \# 2 \# 1 \# 4 \)

Now we simulate
tape 1:

initially

2G #5 #12 #13 #2 #I

4 #7 R

tape 2:

3 #3 #2 #2 #1 #4

tape 3:

tape 4:
to recognize

ACCEPTANCE

-- or Monday--