CPSC 421/501

Midterm: Friday, Nov 3.

Location: In class

- You may use 1 two-sided sheet of notes
- Exam will be 45 minutes long
- You will be seated in alphabetical order; please remain outside until you are asked to come in.
- Exam study guide now available online

Monday, Oct 30 –

20 minutes for midterm

practice questions

Wednesday, Nov 1 –

30 minutes for midterm

practice questions

2023: Same as 2021:

(1) 2023! Decidability & Recognizability

not countable sets (2021)

(2) 2023 some different HW problems

Today: Build a universal Turing machine.

Motivation: to recognize the acceptance problem:

$$\text{ACCEPTANCE}_{\text{Turing machines}} \overset{\text{def}}{=} \left\{ \underbrace{\langle M, i \rangle}_{???} \;\middle|\; \begin{array}{l} M \text{ is a TM,} \\ i \text{ is an input} \\ \text{to } M \text{ s.t.} \\ M \text{ accepts } i \end{array} \right\}$$

Description of a TM, M
and an input, i, to M ...

Formally, a TM is

$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$,

Q is any finite set ☹

Too many finite sets ...

☺ Standardize TM's:

A standard TM is

$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$,

such that:

$$Q = \{1, 2, 3, \ldots, n_Q\}$$

where $n_Q \in \mathbb{N} = \{1, 2, \ldots\}$

$$\Sigma = \{1, 2, 3, \ldots, n_\Sigma\}$$

$$\Gamma = \{1, 2, \ldots, n_\Sigma, n_\Sigma + 1, \ldots, n_\Gamma\}$$

where $n_\Sigma \in \mathbb{N}$

" $n_\Gamma \in \mathbb{N}$ s.t. $n_\Gamma \geq n_\Sigma + 1$

And: $q_0 = 1$, $q_{accept} = 2$, $q_{reject} = 3$

$(n_Q \geq 3)$, blank symbol $= n_\Sigma + 1$

$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject}),$

$\{1, \dots, n_Q\}$ $\quad$ $q_0 \to 1$, $\quad q_{accept} \to 2$, $\quad q_{reject} \to 3$

$\{1, \dots n_\Sigma\}$

$\{1, \dots, n_\Gamma\}$

Describe this TM: we can give

n_Q, n_Σ, n_Γ and description of δ

e.g. $\quad n_Q = 20, n_\Sigma = 5, n_\Gamma = 10$

20 # 5 # 10 # describe δ

(so far we use symbols 0, 1, 2, ..., 9, #

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

for 1-tape TM

$\forall \; q \in \{1, \ldots, n_Q\}$

$\gamma \in \{1, \ldots, n_\Gamma\}$

we need to describe $\delta(q, \gamma)$

$\delta :$ $\delta(1,1), \delta(1,2), \ldots, \delta(1, n_\Gamma)$

$\delta(2,1) \quad \delta(2,2) \quad \ldots$

$\vdots$

$\delta(n_Q, 1) \quad \ldots \quad \delta(n_Q, n_\Gamma)$

Maybe:

$$\underbrace{\delta(1,1)}_{} \# \delta(1,2) \# \ldots \# \delta(n_Q, n_\Gamma)$$

some integer in $\{1, \ldots, n_Q\}$ some tape symbol in $\{1, \ldots, n_\Sigma\}$ L or R

e.g.

$3 \# 2 \# \begin{matrix} L \\ \text{or} \\ R \end{matrix} \#\#$

Alphabet for TM: $\{0, \ldots, 9, L, R, \#\}$

$\langle M \rangle$ if M is standard TM

looks like

$$20\#6\#10\#\overbrace{3\#9\#L\#}^{\delta(1,1)}$$

$$\delta(1,2) \longrightarrow 19\#3\#R\#$$

$$\vdots$$

$\langle M, i \rangle = \langle M \rangle$ then $\langle i \rangle$

$i \in \Sigma^* = \{1, \ldots, n_\Sigma\}^*$

e.g. $\Sigma = \{1, \ldots, 6\}$

$5\#4\#1\#1\#3$

Say $\Sigma = \{1, \ldots, n_\Sigma\}$

and you have a TM

that recognizes

$$L \subset \Sigma^*$$

$$TM = (Q, \Sigma, \Gamma, \ldots)$$

Is there a standard TM

recognizes L ?

Say $Q = \{q_0, q_{acc}, q_{rej}, q_5, q_{mod\,3}\}$

$$\begin{array}{ccccc} q_0 & q_{acc} & q_{rej} & q_5 & q_{mod\,3} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 5 & 4 \end{array}$$

Take

$$\Gamma \supseteq \quad \Sigma \cup \{\sqcup\} \cup \{\#, a, b\}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\{1, \dots, n_\Sigma, \quad n_\Sigma + 1 \quad n_\Sigma + 2, +3, +4$$

=

Again!

Say TM for $\Sigma = \{a, b, c\}$

$$\downarrow \; \downarrow \; \downarrow$$

no harm! $\Sigma = \{1, 2, 3\}$

Now ... build a universal TM!

$\langle M \rangle$ followed by $\langle i \rangle$

standardized

$$\left.\begin{array}{l} 26\#5\#12\#.3\#2\#L \\ \qquad\qquad \#4\#7\#R \\ \qquad\qquad \# \\ \qquad\qquad\quad \vdots \\ \qquad\qquad \#9\#1\#L \end{array}\right\} \langle M \rangle$$

$\langle i \rangle \longrightarrow \#3\#3\#2\#2\#1\#4$

Now we simulate

tape 1 :

initially

2 6 # 5 # 1 2 # 1 3 # 2 # L

4 # 7 # R

#

⋮

9 # 1 # L

3 # 3 # 2 # 2 # 1 # 4 ⊔

⊔ ⊔ ⊔ ...

tape 2 : ⊔ ⊔ ...

tape 3 :

tape 4 :

to recognize

ACCEPTANCE Tm . . .

. . . on Monday . .