

CPSC 421 / 501

Midterm: Friday, Nov 3.

Location: In class

- You may use 1 two-sided sheet of notes
- Exam will be 45 minutes long
- You will be seated in alphabetical order; please remain outside until you are asked to come in.
- Exam study guide now available online

Monday, Oct 30 -

20 minutes for midterm
practice questions

Wednesday, Nov 1 -

30 minutes for midterm
practice questions

2023: Same as 2021:

(1) 2023: Decidability & Recognizability

not countable sets (2021)

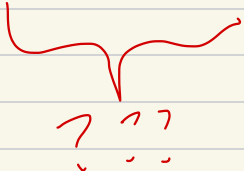
(2) 2023 some different HW problems

Today: Build a universal Turing machine.

Motivation: to recognize the acceptance problem:

ACCEPTANCE Turing machines


def $\{ \langle M, i \rangle \mid \left. \begin{array}{l} M \text{ is a TM,} \\ i \text{ is an input} \\ \text{to } M \text{ s.t.} \\ M \text{ accepts } i \end{array} \right\}$




Description of a TM, M
and an input, i , to M ...

Formally, a TM is

$(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$,

Q is any finite set 

Too many finite sets ...

 Standardize TM's :

A standard TM is

$(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$,

such that :

$$Q = \{1, 2, 3, \dots, n_Q\}$$

$$\text{where } n_Q \in \mathbb{N} = \{1, 2, \dots\}$$

$$\Sigma = \{1, 2, 3, \dots, n_\Sigma\}$$

$$\Gamma = \{1, 2, \dots, n_\Sigma, n_\Sigma + 1, \dots, n_\Gamma\}$$

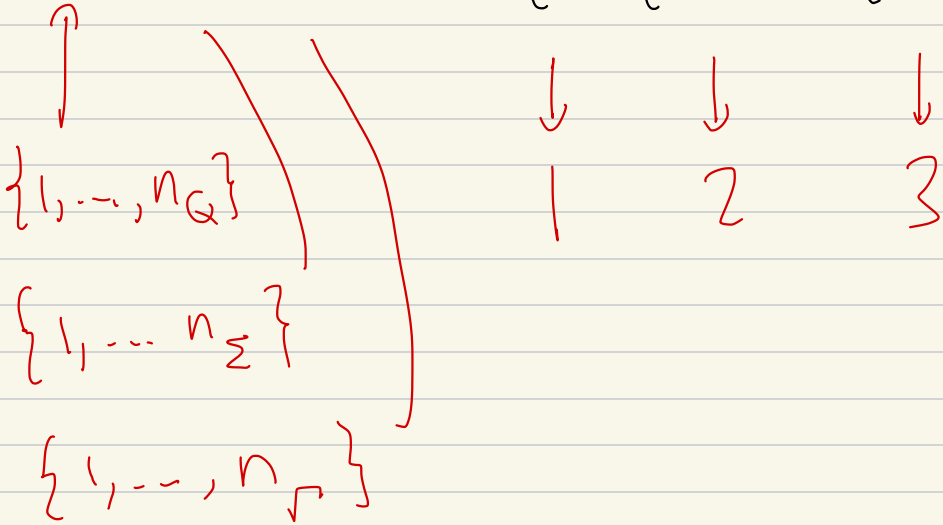
where $n_\Sigma \in \mathbb{N}$

$$\text{" } n_\Gamma \in \mathbb{N} \text{ s.t. } n_\Gamma \geq n_\Sigma + 1$$

$$\text{And: } q_0 = 1, q_{\text{accept}} = 2, q_{\text{reject}} = 3$$

$$(n_Q \geq 3), \text{ blank symbol} = n_\Sigma + 1$$

$(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$,



Describe this TM: we can give

n_Q, n_Σ, n_Γ and description of δ

e.g. $n_Q = 20, n_\Sigma = 5, n_\Gamma = 10$

20 # 5 # 10 # describe δ

(so far we use symbols 0, 1, 2, ..., 9, #)

$$\delta: \mathbb{Q} \times \Gamma \rightarrow \mathbb{Q} \times \Gamma \times \{L, R\}$$

for 1-type TM

$$\forall q \in \{1, \dots, n_{\mathbb{Q}}\}$$

$$\gamma \in \{1, \dots, n_{\Gamma}\}$$

we need to describe $\delta(q, \gamma)$

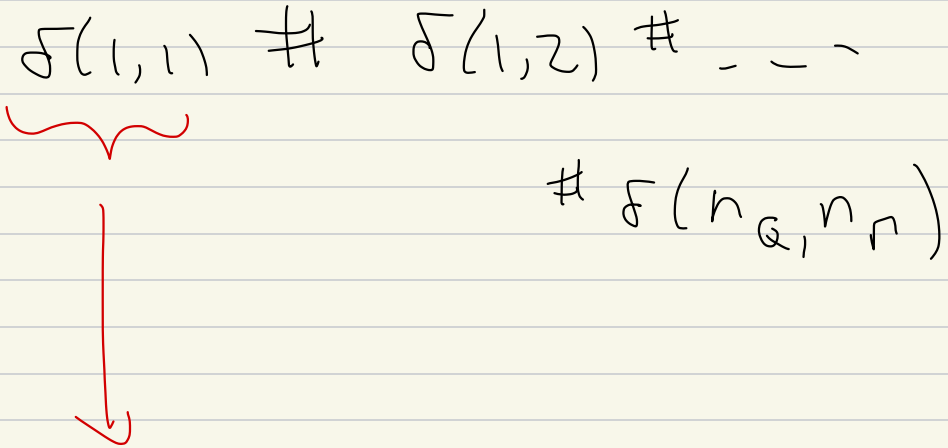
$$\delta: \delta(1, 1), \delta(1, 2), \dots, \delta(1, n_{\Gamma})$$

$$\delta(2, 1) \quad \delta(2, 2) \quad \dots$$

⋮

$$\delta(n_{\mathbb{Q}}, 1) \quad \dots \quad \delta(n_{\mathbb{Q}}, n_{\Gamma})$$

Maybe!



some integer in some tape symbol in L or R

$\{1, \dots, n_Q\}$ $\{1, \dots, n_R\}$

e.g.,

3 # 2 # $\begin{matrix} L \\ \text{or} \\ R \end{matrix}$ ##

Alphabet for TM: $\{0, \dots, 9, L, R, \#\}$

$\langle m \rangle$ if m is standard TM

looks like

$20 \# 6 \# 10 \# \overbrace{3 \# 9 \# L}^{\delta(1,1)} \#$

$\delta(1,2) \rightarrow$

$19 \# 3 \# R \#$
 \vdots
 \vdots

$\langle m, i \rangle = \langle m \rangle$ then $\langle i \rangle$

$i \in \Sigma^* = \{1, \dots, n_\Sigma\}^*$

e.g. $\Sigma = \{1, \dots, 6\}$

$5 \# 4 \# 1 \# 1 \# 3$

Say $\Sigma = \{1, \dots, n_\Sigma\}$

and you have a TM

that recognizes

$$L \subset \Sigma^*$$

$$TM = (Q, \Sigma, \Gamma, \dots)$$

Is there a standard TM

recognizes L ?

$$\text{Say } Q = \{q_0, q_{acc}, q_{rej}, q_5, q_{mod 3}\}$$

↓	↓	↓	↓	↓
1	2	3	5	4

Take

$$\Gamma \ni \Sigma \cup \{\cup\} \cup \{\#, a, b\}$$

↓

↓

↓

$$\{1, \dots, n_\Sigma, n_\Sigma + 1, n_\Sigma + 2, \dots, n_\Sigma + 4\}$$

=

Again!

Say TM for $\Sigma = \{a, b, c\}$

↓ ↓ ↓

no harm! $\Sigma = \{1, 2, 3\}$

Now --- build a universal TM:

$\langle M \rangle$ followed by $\langle i \rangle$

standardized

20 # 5 # 12 # 3 # 2 # 2

4 # 7 # R

i

9 # 1 # 2

} $\langle M \rangle$

$\langle i \rangle \rightarrow$ # 3 # 3 # 2 # 2 # 1 # 4

Now we simulate

tape 1 :

initially

2 | 0 | # | 5 | # | 12 | # | 3 | # | 2 | # | L

→ # 4 # 7 # R

i

9 # 1 # L

→ # | 3 | # | 3 | # | 2 | # | 2 | # | 1 | # | 4 | L

U | U | U | ...

tape 2 : U | U | U | ...

tape 3 :

tape 4 :

to recognize

ACCEPTANCE

TM

or

Munday