CPSC 421/501 Oct 23, 2023

- Multi-tape TM's and

(1) $\{ (i,j,k) \in \mathbb{N}^3 \mid k = i + j \}$

(2) $\{ (i,j,k) \in \mathbb{N}^3 \mid k = i \cdot j \}$

(3) Universal TM

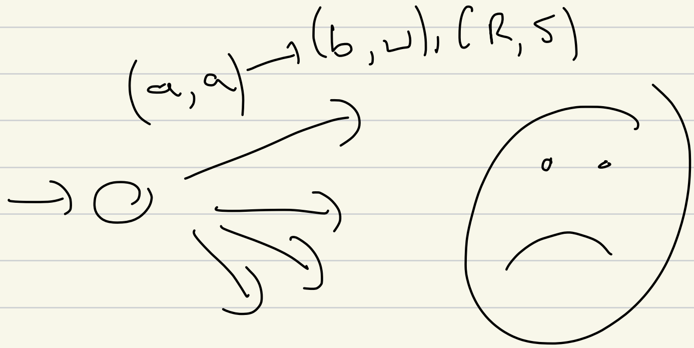
Recall, a universal TM recognizes

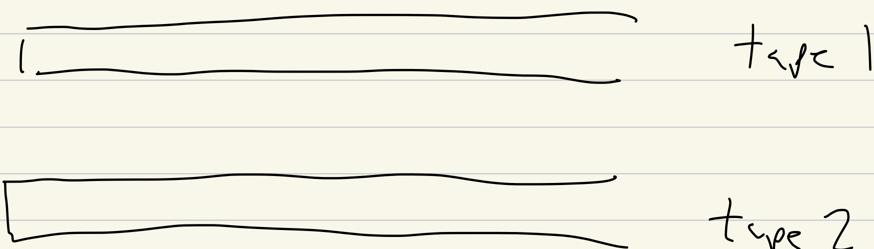
$$\mathrm{ACCEPTANCE}_{TM}$$

$$\stackrel{\text{def}}{=} \{ \langle M, i \rangle \mid M \text{ is a T.M. that accepts the input } i \}$$

Why not give some explicit 2-tape TM algorithms?

They get technical:





Instead...

Say: given

(i, j, k) say base 10

37126 # 48512 # 20159831

Want to know

① is $i + j = k$?

② is $i \cdot j = k$

② is $i \cdot j = k$

37126 ← tape 2

× 48612 ← tape 3

74252 ← tape 4

⋮

← keep a running total

1 is $i + j = k$

37126 ← tape 2

+ 48512 ← tape 3

85638 ← tape 4

i, j = 2

```
       37126
×      48512
------------
       74252   ←   tape 4
      37126    ←   "  4
    185630     ←   "  4
```

↓

running total ← tape 5
sum

Multi-tape TM:

Makes it easier (more convincing) to describe algorithms, closer to actual implementation.

Rules:

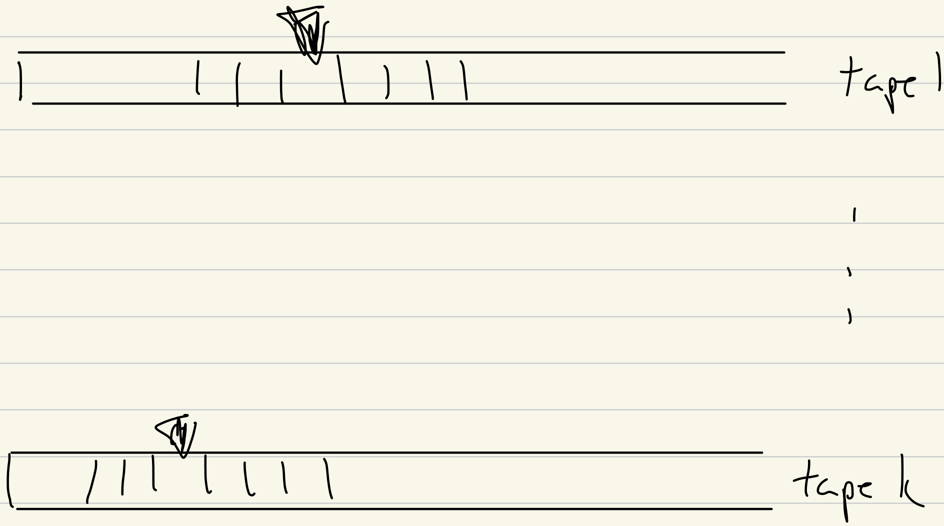
- finite number of tapes (independent of the input, say k tapes, $k = 1, 2, 3, \ldots$

Then k-tape machine

$$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$$

only difference

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R, S\}^k$$



<u>Thm</u>: Any k-tape TM, M, has an <u>equivalent</u> 1-tape TM, M'.

Equivalent means

1. literally, for all inputs i,

M accepts i $\Leftrightarrow$ M' accepts i

M reject i $\Leftrightarrow$ M' rejects i

M loops on i $\Leftrightarrow$ M' loops on i

never halts

Q: What is the difference in speed?

Ans: PALINDROME requires n^2 time (for an input of length n) on a 1-tape TM, but $O(n)$ time on a 2-tape.

Claim: Any algorithm on a k-tape machine, that runs in time

$\leq f(n)$, where $f(n) = n^{\alpha}$

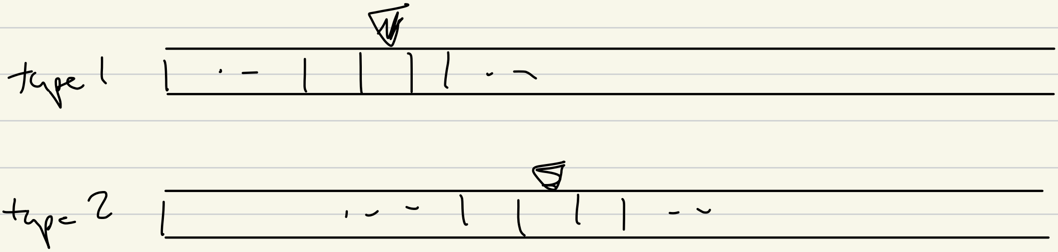
some $\alpha > 1$, can be run in time

$O((f(n))^2) = O(n^{2\alpha})$ on

a 1-tape machine.

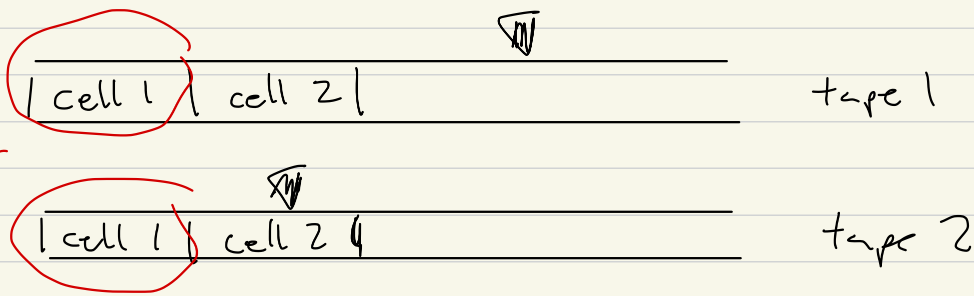
Proof idea:

Given 2-tape algorithm. How to simulate on 1-tape machine

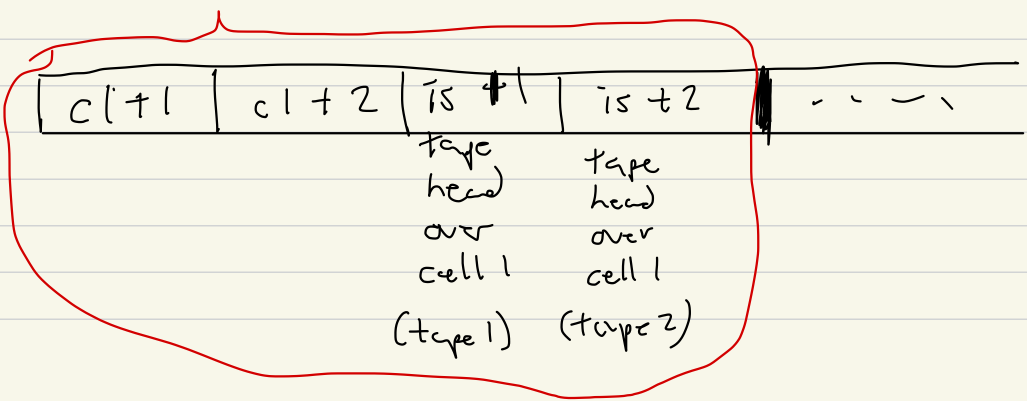


Each cell has some element of Γ

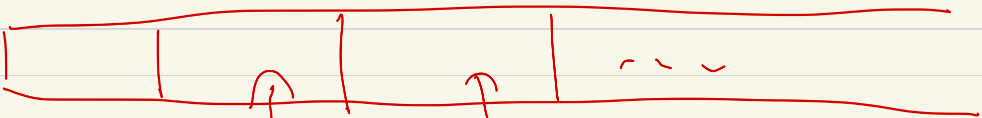
So: let's combine the tapes:



by intertwining



1-tape equivalent

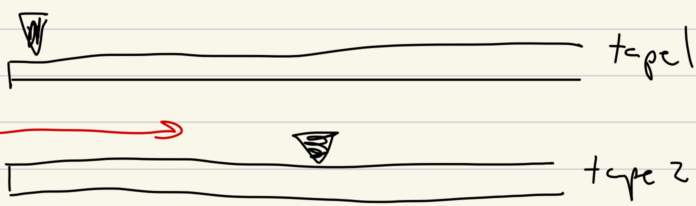


stuff about cell 1 tapes 1,2 of 2-tape algorithm

same about cell 2

same about cell 3

Rem: After time T of a 2-tape alg., i.e. T steps of the algorithm could have



on cell T

This accounts for possibly time on 1-tape machine, for T steps on 2-tape machine, to have total time

$$\text{order}\left(1+2+3+\ldots+T\right)$$

$$= \text{order}\left(\binom{T+1}{2}\right)$$

$$= \text{order}\left(T^2\right)$$

(going from time T to order(T^2) is essentially optimal, see CPSC 50)

Alternate way to go from

2-tapes to 1-tape algorithm:

[Sip]

1-tape

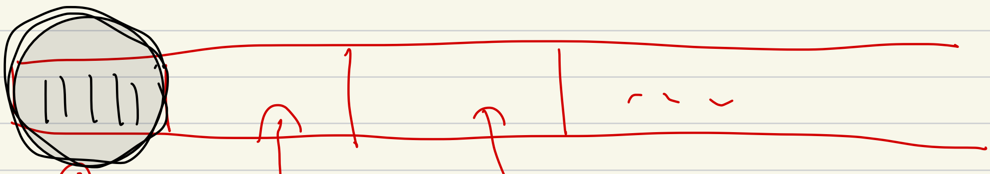
write all tape 1 info | tape 2 info

You may need T tape

cells at time T

Alternate, variant of method 1

1-tape equivalent



stuff about cell 1 tapes 1,2 of ~~2-tape~~ 2-tape algorithm

same about cell 2

same about cell 3

could go from Γ to

to $\Gamma_{big} = \Gamma \times \Gamma \times \left\{ \begin{matrix} \text{is tape head} \\ \text{here?} \end{matrix} \right\}^2$

Also: for input

σ_1	σ_2	$\dots$	σ_n	$\sqcup$	$\sqcup$	$\sqcup$

add

$$\Sigma \text{ to } \Gamma_{big}$$

so that

$$\Sigma \subset \Gamma_{big}$$

Friday:

Universal TM's ...