CPSC 421/501 Oct 13, 2023

Myhill-Nerode: L, Σ:

$$\mathrm{AccFut}_L(s) \overset{\text{def}}{=} \{ t \in \Sigma^* \mid st \in L \}.$$

L and L^{reverse}

If $w = \sigma_1 \ldots \sigma_n$ is a string over Σ, length n,

$$w^{\text{rev}} = \sigma_n \sigma_{n-1} \ldots \sigma_1$$

Last time:

DIV-BY-2

$= \{0, 2, 4, 6, 8, 10, 12, \ldots\}$

$$\boxed{L^{reverse} \overset{def}{=} \{ w^{rev} \mid w \in L \}}$$

$(\text{DIV-BY-2})^{reverse}$

$= \{0^{rev}, 2^{rev}, \ldots, 8^{rev}, 10^{rev}, 12^{rev}, \ldots\}$

$= \{0, 2, \ldots, 8, 01, 21, 41, \ldots\}$

$= \{ 0, 2, 4, 6, 8, 01, 02, 03, \ldots, 09,$

$21, 22, 23, \ldots, 29,$

$41, 42, \ldots \quad \ldots \}$

$=$

DIV-BY-2-LEAST-SIG-FIRST-NO-LEADING-0s-ALW

$=$ DIV-BY-2^{rev}

$=$ 2-YB-VID $=$ 2-YB-VID ??

$=$

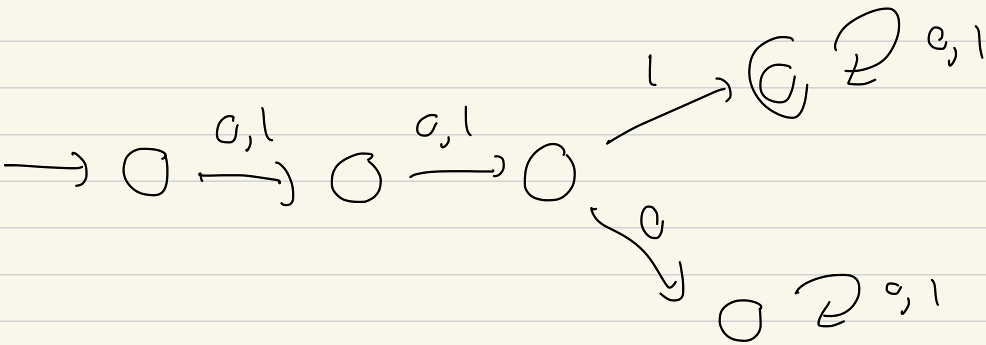
Upshot:

$L_k = \{ w \in \{0,1\}^* \mid$ the k-th symbol of w is 1 $\}$

$L_3 = \{ 001, 011, 101, 111,$

$0010, 0011, \ldots \}$

DFA



$(L_k)^{rev}$ homework:

we'll see that there is

an NFA roughly k states

$k + O(1)$

DFA for L_k^{rev}

requires $c \cdot 2^k$ states

(Myhill-Nerode) (we'll see)

$(L^{rev})^{rev} \stackrel{?}{=} L$

Myhill-Nerode:

Given L language over Σ,

$$\mathrm{AccFut}_L(s) = \{ t \in \Sigma^* \mid st \in L \}$$

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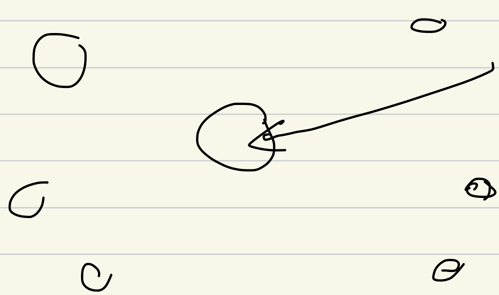
DFA for L



s_1, s_2 taken to same state

Then

$$\mathrm{AccFut}_L(s_1) = \mathrm{AccFut}_L(s_2).$$

So:

$$L = \{ 0^n 1^n \mid n = 0, 1, \ldots \}$$

$$= \{ \varepsilon, 01, 0^2 1^2, 0^3 1^3, \ldots \}$$

$$\mathrm{AccFut}_L(\varepsilon) = L = \{\varepsilon, 01, 0^2 1^2, \ldots\}$$

$$\mathrm{AccFut}_L(1) = \emptyset$$

$$\mathrm{AccFut}_L(11011001) = \emptyset$$

$\vdots$

$$\mathrm{AccFut}_L(0) = \{1, 011, 0^2 1^3, \ldots\}$$

$$\mathrm{AccFut}_L(00) = \{1^2, 01^3, 0^2 1^4, \ldots\}$$

$\vdots$

$$\mathrm{AccFut}_L(0^k) = \{1^k, 01^{k+1}, \ldots\}$$

So if $k_1 \neq k_2$ $k_1, k_2 \in \mathbb{N}$

then

$$1^{k_1} \in \text{Accfut}_L(0^{k_1})$$

$$1^{k_1} \notin \text{Accfut}_L(0^{k_2})$$

or, more simply

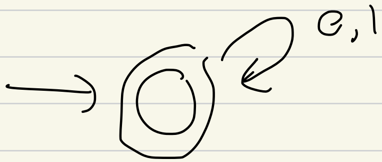
$$0^{k_1} 1^{k_1} \in L$$

$$0^{k_2} 1^{k_1} \notin L$$

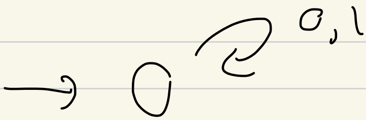
So $0, 0^2, 0^3, 0^4, \ldots$ need to be in different states of a DFA recognizing L.

Last time: simplest DFA's

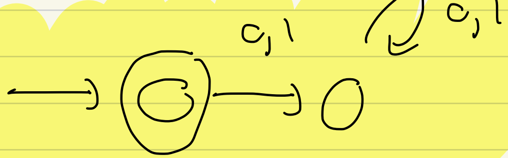
$\Sigma = \{0,1\}$



recognizes $\{0,1\}^*$



recognizes $\emptyset$



recognizes $\{\varepsilon\}$

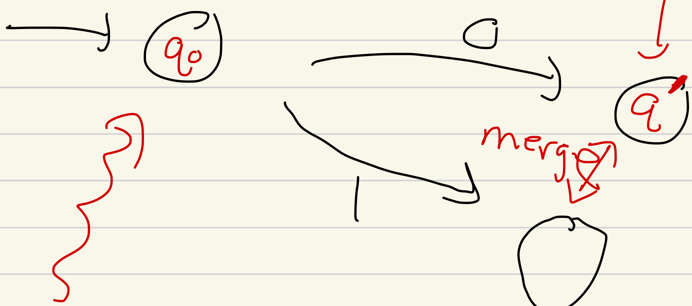
$$L = \{\varepsilon\}$$

$$\boxed{\mathrm{AccFut}_{\{\varepsilon\}=L}(\varepsilon) = \{\varepsilon\}}$$

$$\mathrm{AccFut}_{\{\varepsilon\}=L}\left(\begin{array}{c}\text{anything}\\ \text{else}\end{array}\right) = \emptyset$$

AccFut = $\{\varepsilon\}$

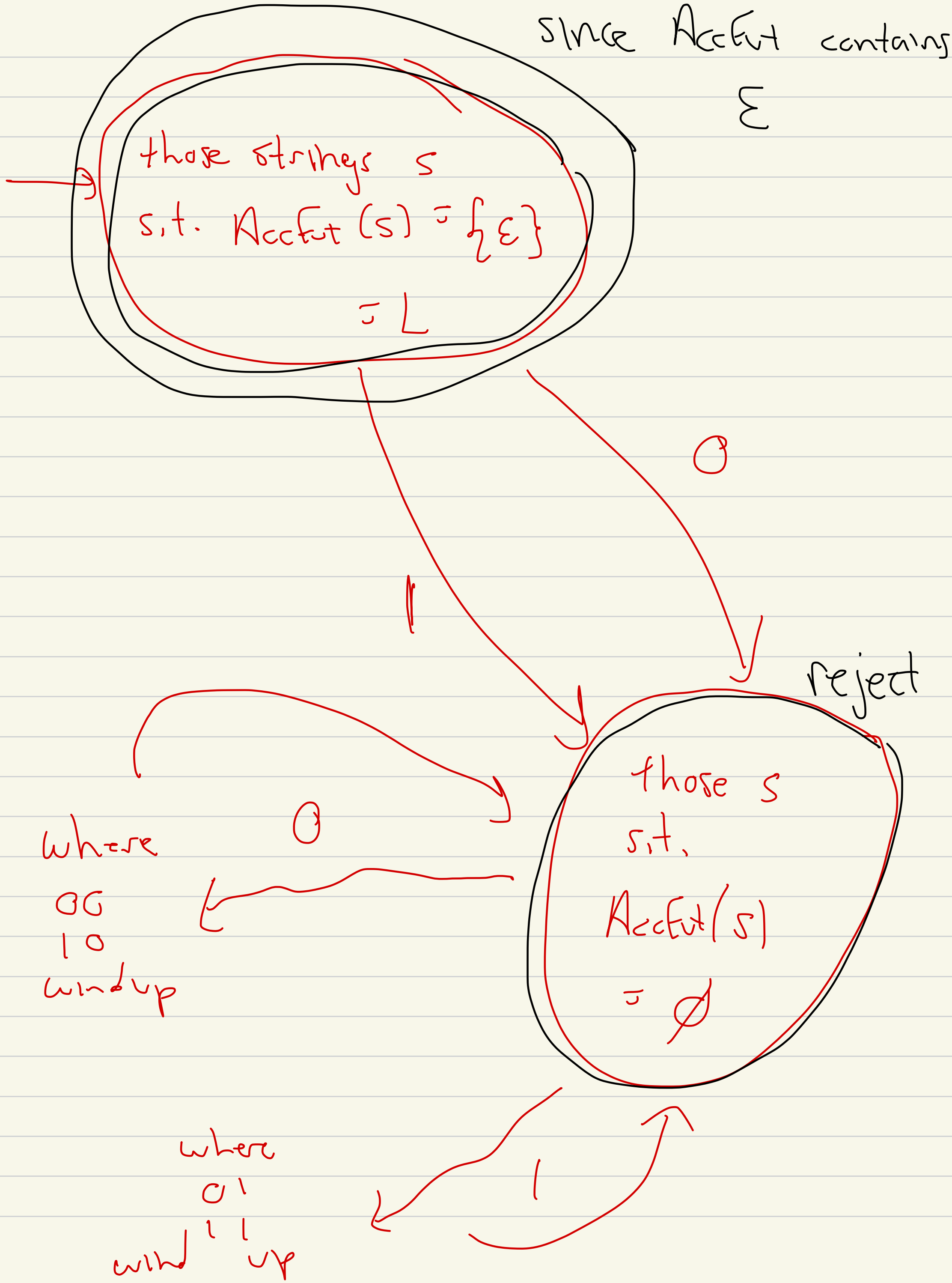
"0" is taken here



ε is taken to initial state

$\mathrm{AccFut}(0) = \emptyset$

$\mathrm{AccFut}(1) = \emptyset$



since AccFut contains
ε
those strings s
s.t. AccFut(s) = {ε}
= L
1
0
reject
those s
s.t.
AccFut(s)
= ∅
0
where
00
10
wind up
1
where
01
11
wind up

DIV-BY-3 $\subset \{0,1\}^*$

strings in $\{0,1\}^*$ s.t. in base 10, the string represents an integer divisible by 3

1 0 1 1 0 1 1 1 0 1 1 1 1

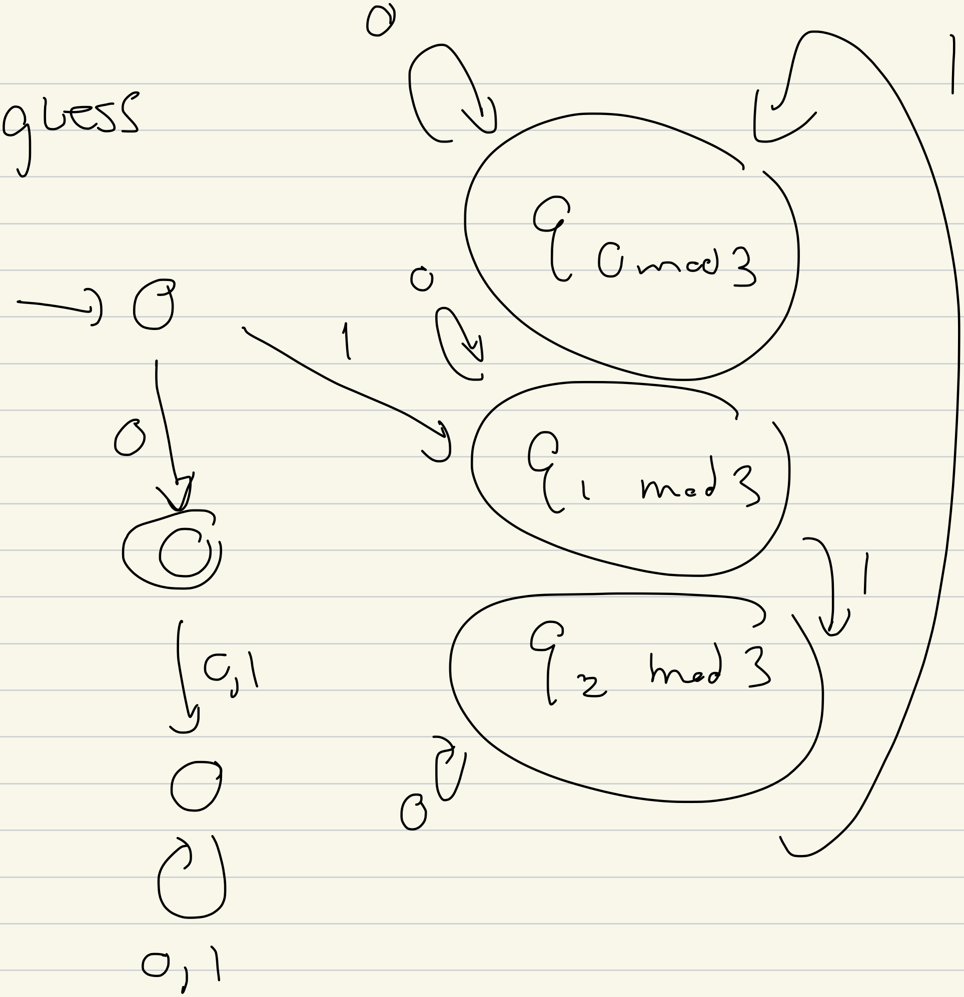
is sum: $\equiv 0 \pmod 3$

$\equiv 1 \pmod 3$

$\equiv 2 \pmod 2$

DIV-BY-3 = {0, 111, 1011, 1101, 1110, ...}

guess



$\mathrm{AccFut}_L(s)$:

$s = \varepsilon$, $\mathrm{AccFut}(\varepsilon) = L$

$= L = \{0, 111,$

$s = 0$, $\mathrm{AccFut}(0) = \{\varepsilon\}$

$\mathrm{AccFut}(1) = \{11, 011, 101, \ldots\}$

$= \{ s \mid s$ contains # of 1's that is $\equiv 2 \mod 3 \}$

Accfut = L