CPSC 421/501

Thursday, October 12, 2023, virtual Monday

- A regular expression for:

  DIV-BY-3 = {0, 3, 6, ...}

  a cautionary tale ...

- Myhill-Nerode Theorem

  In principle (1) determines the min # of states in a DFA that recognizes a language (2) determines non-regularity

Last time:

If L is described by

a regular expression, then

we can build

an NFA that recognizes L,

a DFA " " L,

so L is regular.

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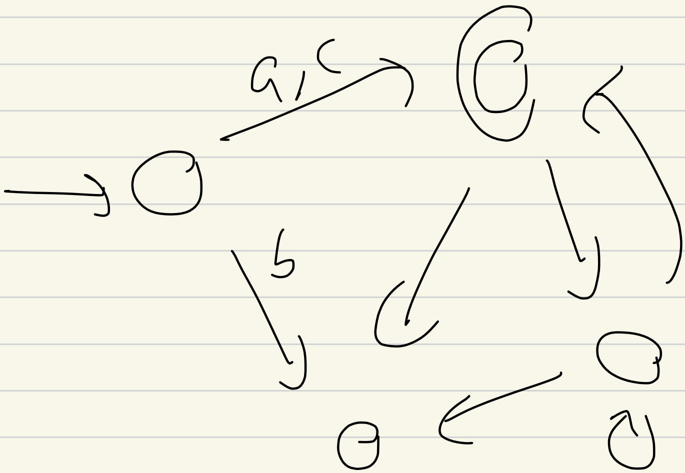
Thm: Any regular language, L,

is described by some regular expression

Idea:



etc.

say s_1, s_2 both wind up in

=

Given $L \subset \Sigma^*$, for each $s \in \Sigma^*$

define

$$\text{AcceptingFutures}_L(s)$$
$$= \{ t \in \Sigma^* \mid st \in L \}$$

$$\text{DIV-BY-2} \subset \Sigma^*_{0,\ldots,9}$$

$$= \{0, \ldots, 9\}^*$$

$$= \{0, 2, 4, \ldots\}$$

$$\text{DIV-BY-2-LEADING-0's-ALLOWED}$$

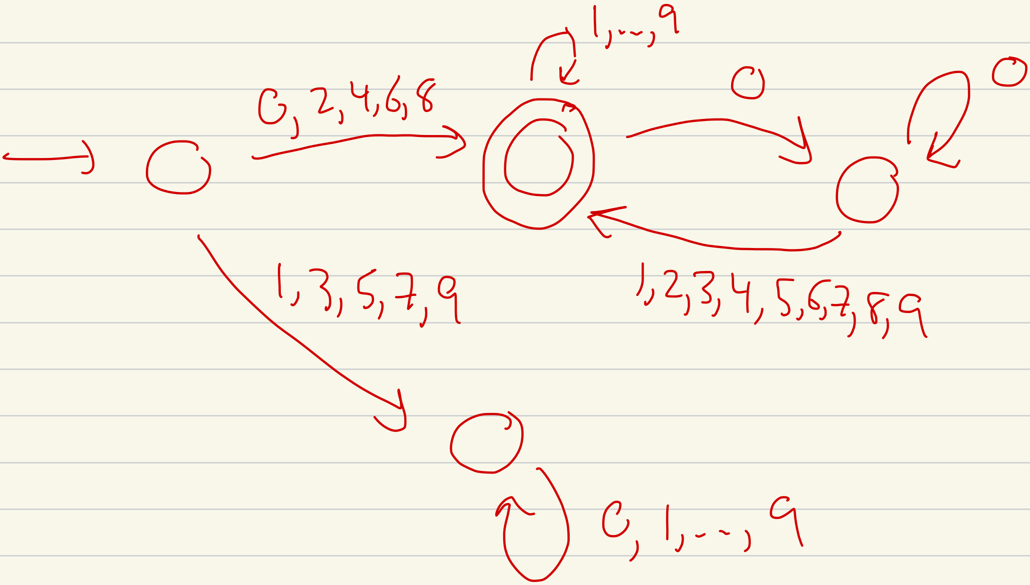
$$= \{0, 2, 4, 6, 8, 00, 02, \ldots\}$$

$$\text{DIV-BY-2-LEAST-SIG-FIRST-NO-LEADING-0's-ALLW}$$

$$= \{0, 2, 4, 6, 8, 01, 21, 41, 61, 81, 02, 22, 42, \ldots\}$$

00 not in

DIV-BY-2-LEAST-—



Can we have fewer states?

How would we know???

Example of a non-regular language

is

$$L = \{ 0^n 1^n \mid n = 0, 1, \ldots \}$$

$$= \{ \varepsilon, 01, 0011, 000111, 0^4 1^4, 0^5 1^5, \ldots \}$$

Intuitively: We have to scan input

$$0000 \ldots 000 \ldots 00$$

and we have only finitely many states, ...

$\mathrm{Accfut}_L(00)$

$$= \{ \sigma_1 \ldots \sigma_m \text{ such that } 00\,\sigma_1\sigma_2\ldots\sigma_m \in L \}$$

$$= \{ 11,\ 0111,\ 001111,\ \ldots \}$$

$0011 \in L$

$001111 \in L$

$$L = \{\varepsilon, 01, 0011, \ldots\}$$

$$\mathrm{AccFut}_L(10)$$

$$= \left\{ t \in \{0,1\}^* \;\middle|\; \begin{array}{l} t = \sigma_1 \ldots \sigma_m \\ 10t \\ = 10\sigma_1\sigma_2\ldots\sigma_m \\ \in L \end{array} \right\}$$

$$= \emptyset$$

$$\mathrm{AccFut}_L(\varepsilon) = \{ t \mid \underbrace{\varepsilon t}_{t \in L} \in L \}$$

$$= L$$

$$\mathrm{AccFut}_L(0) = \{1,\ 011,\ 0^2 1^3, \ldots\}$$

$$\mathrm{AccFut}_L(00) = \{1^2,\ 01^3, \ldots\}$$

$$\mathrm{AccFut}_L(000) = \{1^3,\ 01^4,\ 0^2 1^5, \ldots\}$$

$$\vdots$$

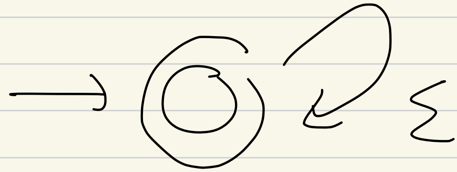
$$\mathrm{AccFut}_L(0^m) = \{1^m,\ 01^{m+1}, \ldots\}$$

$$\vdots$$

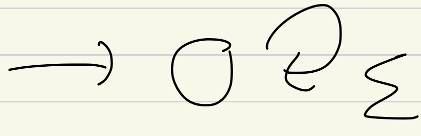
all of these are different

so no DFA can recognize L

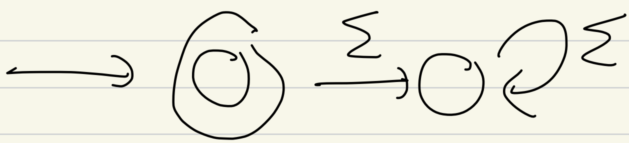
$L = \Sigma^*$



$L = \emptyset$



$L = \{\varepsilon\}$



$\Sigma = \{a, b\}$

$L = \{\varepsilon\}$

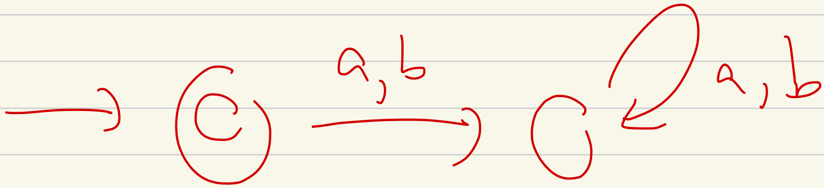
$$\mathrm{AccFut}_{\{\varepsilon\}}(abaacab) = \emptyset$$

$$\mathrm{AccFut}_{\{\varepsilon\}}(abba) = \emptyset$$

$$\mathrm{AccFut}_{\{\varepsilon\}}(a) = \emptyset$$

$$\mathrm{AccFut}_{L=\{\varepsilon\}}(\varepsilon) = \{\varepsilon\}$$

Class ends ...



DFA $\uparrow$ no ε-transitions