A regular expression for:

\[ \text{DIV-BY-3} = \{ 0, 3, 6, \ldots \} \]

a cautionary tale ... 

- Myhill-Nerode Theorem

In principle (1) determines the min # of states in a DFA that recognizes a language (2) determines non-regularity
Last time:

If $L$ is described by a regular expression, then we can build an NFA that recognizes $L$, a DFA \( \equiv \) $L$, so $L$ is regular.

Thm! Any regular language, $L$, is described by some regular expression.
Idea:

Given \( L \subseteq \Sigma^* \), for each \( s \in \Sigma^* \) define

\[
\text{Accepting Futures}_L(s) = \{ t \in \Sigma^* \mid s.t \in L \}
\]
$$\text{DIV-BY-2} < \sum_{0,\ldots,9} \cdot \{c, \ldots, a\}^* = \{0, 2, 4, \ldots\}$$

DIV-BY-2 - LEADING 0'S - ALLOWED

$$\text{DIV-BY-2} - \text{LEAST SIG - FIRST - NO-LEADING - 0'S - ALL}$$

$$\{0, 2, 4, 6, 8, 00, 02, \ldots\}$$

DIV-BY-2 - LEAST SIG - FIRST - NO-LEADING - 0'S - ALL

$$\{0, 2, 4, 6, 8, 01, 21, 41, 61, 81, 02, 22, 42, \ldots\}$$
Can we have fewer states?
How would we know???
Example of a non-regular language is

\[ L = \{ 0^n 1^n \mid n = 0, 1, 2, \ldots \} \]

\[ = \{ \varepsilon, 01, 0011, 000111, 0^4 1^4, 0^5 1^5, \ldots \} \]

Intuitively! We have to scan input

\[ \underbrace{000 \ldots 000}_{\text{many}} \]

and we have only finitely many states, \ldots
\[ \text{Acc} \text{full}_L^+ (\Sigma_0) \]

\[ = \left\{ \sigma_1 \cdots \sigma_m \text{ such that } \right. \\
\left. \sigma_1 \sigma_2 \cdots \sigma_m \in L \right\} \]

\[ = \left\{ \{ \lll, \lllll \}, \lllllll, \ldots \right\} \]
\[ L = \{ \varepsilon, 01, 0011, \ldots \} \]

\[ \text{Acc} \cap (10) \]

\[ = \left\{ t \in \{0,1\}^* \mid t = 0 \cdot \ldots \cdot 0 \right\} \]

\[ = \{ 0 \sigma_1 \sigma_2 \ldots \sigma_n \mid \sigma \in L \} \]

\[ = \emptyset \]

\[ \text{Acc} \cap (E) \]

\[ = \left\{ t \mid E \vdash L \right\} \]

\[ = L \]
\[ \text{Acc}_L(0) = \{ 1, 01, 0^21^3, \ldots \} \]
\[ \text{Acc}_L(00) = \{ 1^2, 01^3, \ldots \} \]
\[ \text{Acc}_L(000) = \{ 1^3, 01^4, 0^21^5, \ldots \} \]
\[ \text{Acc}_L(0^m) = \{ 1^m, 01^{m+1}, \ldots \} \]

So no DFA can recognize

all of these are different
\[ L = \varepsilon \rightarrow \emptyset \]

\[ L = \emptyset \rightarrow \emptyset \]

\[ L = \{ \varepsilon \} \]

\[ \Sigma = \{ a, b \} \]
\[ L = \{ \varepsilon \} \]

\[ \text{AccFut}_L(\varepsilon) = \emptyset \]

\[ \text{AccFut}_L(\text{abba}) = \emptyset \]

\[ \text{AccFut}_L(\text{a}) = \emptyset \]

\[ \text{AccFut}_L(\varepsilon) = \{ \varepsilon \} \]
Class ends

\[ C \xrightarrow{a,b} C \]

DFA The \( \varepsilon \)-transitions