- NFA's and regular expressions (with $\emptyset, \Sigma, \varepsilon, 0, 1, *$

- Implementing NFA's

- Why we skip the theorem that a DFA/NFA has an equivalent regular expression. (DIV-BY-3)

- Start Myhill-Nerode

(Perhaps tomorrow, on Thursday, virtual Monday)
Reg Exp examples

\[(a^3 \cup a^5)^*\]

\[(a \cup ab)^*\]

6.1.2: 4 points \{ 1 point is extra credit \}

6.1.5: 4 points \{ 2 points are extra credit \}

What are all the possible values of \(\text{Period}(L)\) given that all you know about \(L\) is that …

If \(L\) is not regular, \(\text{Period}(L)\) \{ does not exist \}
There are many notions of regular expressions:

In Unix: \( \text{ls } (-[A,a])^* \)

or something like this ... \( [^Aa] \)

Rem: In [Sip] don't use \( \sim, \wedge \), We limit ourselves to \( \phi \), an element of \( \Sigma \) (underlying alphabet), \( \varepsilon \), \( 0 \), \( 0 \), \( \ast \)
Formally:

\[ \{a^3, a^5\}^* \]

\[ \Sigma = \{a, \ldots \} \]

Start with a

then \((a \circ a)\)

then \((a \circ a) \circ a \leftarrow \{a^3\}\)

\[(a \circ a) \circ a \cup (\overbrace{a \circ \cdots \circ a}^k) \]

in theory
Formally:

(1) $\emptyset$ is a regular expression.

(2) Any element of $\Sigma$ is R.E.

(3) $\varepsilon$ is R.E.

If $R_1, R_2$ are R.E., so are

$(R_1 \cup R_2), (R_1 \circ R_2), (R_1)^*$

We tend to

- omit $\emptyset$

- "any unnecessary ()"
e.g.

\[(a + a) \cdot a = a^* (a + a^* a)\]

usually write \(a^3\)

= Another example \((a + ab)^*\)

technically \(\#(a + (a + b))^{\#}\)

\((a + ab)^*\)

A regular expressions \{ defines \{ describes \}

a language --
e.g.,

\[ \emptyset \text{ describes } \emptyset \]

\[ \Sigma \text{ } \subseteq \text{ } \{ \sigma \} \]

\[ \varepsilon \text{ } \subseteq \text{ } \{ \varepsilon \} \]

If \( R_1, R_2 \) are R.E. describeable \( L_1, L_2 \) then

\( (R_1 \cup R_2) \) describes \( L_1 \cup L_2 \)

\( (R_1)^* \) describes \( L_1^* \)
Thm: A regular expression describes a regular language, and, conversely, every regular language is described by a regular expression.

E.g. \((a\cup ab)^*\)

NFA \((a\cup ab)\)

\[\rightarrow O \xrightarrow{a} \circ \xrightarrow{b} O\]
NFA: 5-tuple \((Q, \Sigma, \delta, q_0, F)\)

\[
\delta : Q \times \Sigma \rightarrow \text{Power}(Q)
\]

\[
\Sigma_{\varepsilon} = \Sigma \cup \{\varepsilon\}
\]

Thm: Let \(M = (Q, \Sigma, \delta, q_0, F)\) be an NFA. Then there is an equivalent NFA \(\hat{M} = (\hat{Q}, \Sigma, \hat{\delta}, \hat{q}_0, \hat{F})\)

\[
\hat{Q} = \text{Power}(Q),
\]

\[
\hat{\delta} : \hat{Q} \times \Sigma \rightarrow \hat{Q}
\]
\[ \tilde{\delta}(\tilde{q}, \sigma) = \{ q \in Q \mid \text{on } \tilde{q}, \ \text{reading } \sigma \} \]\n
and possibly E's, you can transition to \( q \).

\[ \hat{q} \in \hat{Q} = \text{Power}(Q) \]

\[ \hat{q} \in \text{Power}(Q) \]

e.g.,

\[ \begin{align*}
\hat{Q} &= \text{Power}(Q) = \left\{ \emptyset, \{ q_0 \}, \{ q_1 \}, \{ q_2 \}, \{ q_0, q_1 \}, \{ q_0, q_2 \}, \{ q_1, q_2 \} \right\}
\end{align*} \]
Say
\[ \delta \left( \{ q_0, q_2 \}, b \right) = \] (1) fall off the edge of the world
\[ \text{and} \]
(2) "..."

So
\[ \delta \left( \{ q_0, q_2 \}, b \right) = \emptyset \]

Rem:
\[ \delta ( \emptyset, \text{anything} ) = \emptyset \]
What is the initial state of DFA corresponding to NFA?

\[ q_0 \rightarrow 0 \]

\[ \delta(q_0, \varepsilon) = q_0 \]

- \( \varepsilon \)-jump to yourself does nothing

\[ \hat{q}_0 = \{ q \in Q | q_0 \text{ or any state you can reach from } q_0 \text{ with } \varepsilon \text{ jumps} \} \]
\[ F = \{ \bar{q} \in Q \mid \bar{q} \cup F \text{ is non-empty} \} \]

\[ |Q| = 20 \text{ in NFA, the corresponding DFA has state set } \text{Power}(Q) \]

\[ \text{Size } 2^{20} > 1,000,000 \]

NFA

\[ \begin{array}{c}
\text{\( \bar{q}_4 \) } \\
\vdots \\
\text{\( \bar{q}_6 \) }
\end{array} \]

\[ \bar{q}_{19} \]
Say we in DFA state

\[ \{ q_0, q_3, q_5, q_{12}, q_{13}, q_{16} \} = \tilde{\varphi} \]

Here and we read "b"

\[ \delta(\tilde{\varphi}, b) \]

Where can b and \( \varepsilon \)'s take us? from \( \tilde{\varphi} \)

\( \tilde{\varphi} \rightarrow \) is q0? is q1?

\[ 0, 1 \]