CPSC 421/501 Oct 11, 2023

- NFA's and regular expressions (with $\emptyset, \Sigma, \varepsilon, \cup, \circ, *$)

- Implementing NFA's

- Why we skip the theorem that a DFA/NFA has an equivalent regular expression. (DIV-BY-3)

- Start Myhill-Nerode

(Perhaps tomorrow, on Thursday virtual Monday)

$\boxed{\begin{array}{l}\text{– Reg Exp examples} \\ (a^3 \cup a^5)^* \\ (a \cup ab)^* \end{array}}$

6.1.2 : 4 points $\left\{ \begin{array}{l} \text{1 point is} \\ \text{extra credit} \end{array} \right\}$

6.1.5 : 4 points $\left\{ \begin{array}{l} \text{2 points are} \\ \text{extra credit} \end{array} \right\}$

What are all the possible values of Period(L) given that all you know about L is that . . .

<u>If</u> <u>L</u> <u>is</u> <u>not</u> <u>regular</u>, Period(L) $\left\{ \begin{array}{l} \text{does} \\ \text{not} \\ \text{exist} \end{array} \right.$

There are many notions of regular expressions:

In Unix: ls $(\neg[A,a])*$

or something like this ...

[^Aa]

Rem: In [Sip] don't use $\neg$, ^,

We limit ourselves to

$\emptyset$, an element of Σ (underlying alphabet), ε, $\cup$, $\circ$, $*$

Formally:

$$\{a^3, a^5\}^*$$

$$\Sigma = \{a, \ldots\}$$

Start with a

then $(a \circ a)$

then $((a \circ a) \circ a) \leftrightarrow \{a^3\}$

various parethesis

$$\left(((a \circ a) \circ a) \cup (\underbrace{a \circ \ldots \circ a}_{a^5}) \right)^*$$

in theory...

Formally:

(1) $\emptyset$ is a regular expression

(2) any element of Σ is R.E.

(3) ε " " "

If R_1, R_2 are R.E., so are

$$(R_1 \cup R_2), \quad (R_1 \circ R_2), \quad (R_1)^*$$

We tend to

- omit $\circ$
- " any unnecessary (,)

e.g.

$$((a \circ a) \circ a) = (a \circ (a \circ a))$$

usually write a^3

=

Another example $(a \cup ab)^*$

technically $\cancel{\{}(a \cup (a \cancel{\circ} b))\cancel{\}}^*$

$$(a \cup ab)^*$$

A regular expressions $\left\{ \begin{array}{l} \text{defines} \\ \text{describes} \end{array} \right\}$

a language ...

e.g.,

$\emptyset$ describes $\emptyset$

$\sigma \in \Sigma$ " $\{\sigma\}$

ε " $\{\varepsilon\}$

If R_1, R_2 are R.E.

describe L_1, L_2 then

$(R_1 \cup R_2)$ describes $L_1 \cup L_2$

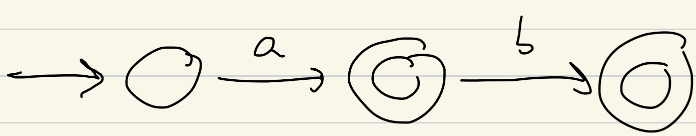
$\circ$ $\circ$

$(R_1)^*$ " L_1^*

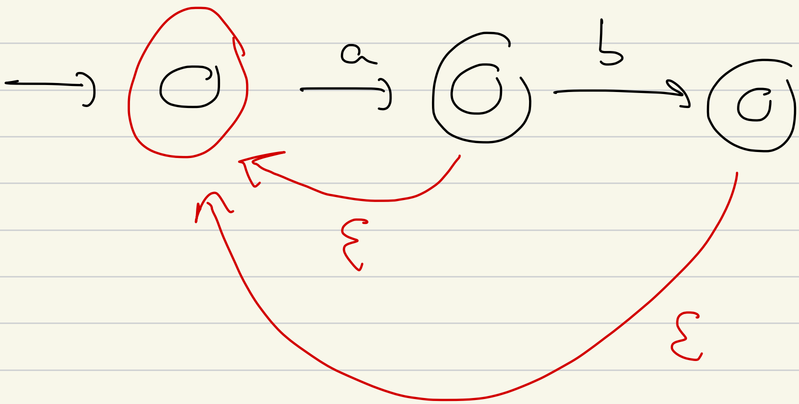
Thm: A regular expression describes a regular language, and, conversely, every regular language is described by a regular expression.

E.g. $(a \cup ab)^*$

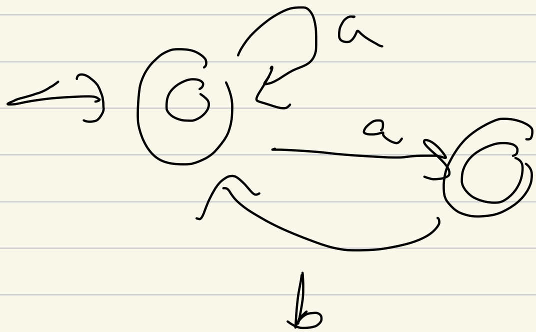
NFA $(a \cup ab)$



NFA $(a \cup ab)^*$



OR



NFA: 5-tuple $(Q, \Sigma, \delta, q_0, F)$

$$\delta : Q \times \Sigma_\varepsilon \to \text{Power}(Q)$$

$$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$

Thm: Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Then there is an equivalent DFA $= (\hat{Q}, \Sigma, \tilde{\delta}, \tilde{q}_0, \tilde{F})$

$\tilde{Q} = \text{Power}(Q)$,

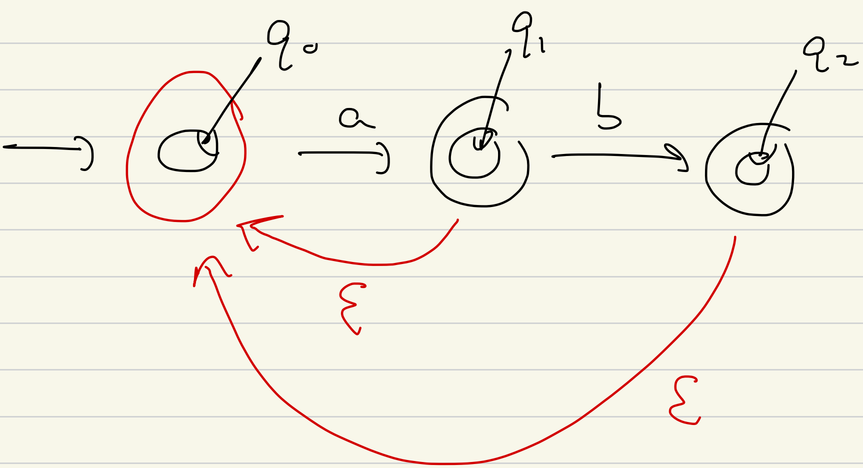
$$\tilde{\delta} : \tilde{Q} \times \Sigma \to \tilde{Q}$$

$$\tilde{\delta}(\tilde{q}, \sigma) = \{ q \in Q \mid \text{on } \tilde{q}, \text{ reading } \sigma \text{ and possibly } \varepsilon\text{'s, you can transition to } q \}$$

$$\tilde{q} \in \tilde{Q} = \text{Power}(Q)$$

$$\tilde{q} \subset \text{Power}(Q)$$

e.g.



$$\tilde{Q} = \text{Power}(Q) = \{ \emptyset, \{q_0\}, \{q_1\}, \{q_2\}, \{q_1, q_2\}, \{q_1, q_0\}, \{q_2, q_0\}, \{q_0, q_1, q_2\} \}$$

Say

$$\delta(\{q_0, q_2\}, b)$$

$q_0 \xrightarrow{b}$ fall off the edge of the world

$q_2 \xrightarrow{b}$ " " " " " " "

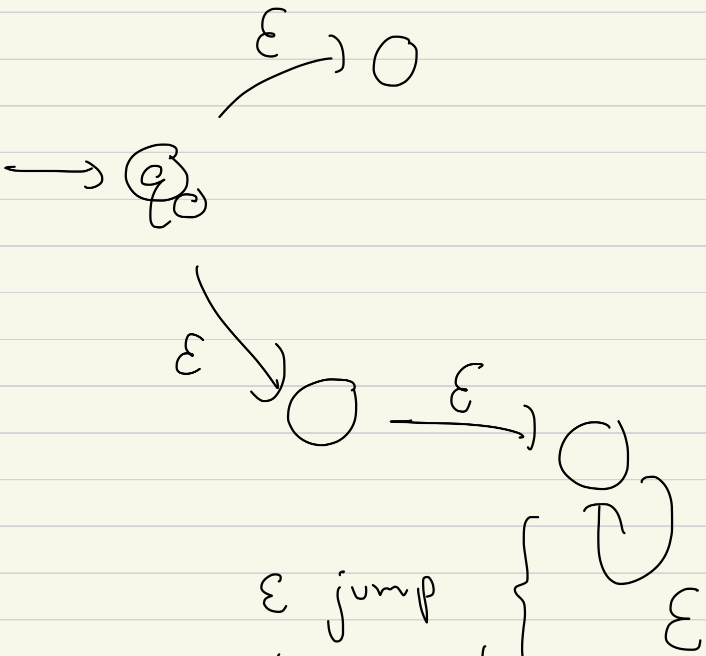
So

$$\delta(\{q_0, q_2\}, b) = \emptyset$$

Rem:

$$\delta(\emptyset, \text{anything}) = \emptyset$$

What $\tilde{q}_0$ initial state of DFA corresponding to NFA?



ε jump to yourself does nothing

$\tilde{q}_0 = \{ q \in Q \mid q_0$ or any state you can reach from q_0 with ε jumps $\}$

$$\tilde{F} = \{ \tilde{q} \subset Q \mid \tilde{q} \cap F \text{ is non-empty} \}$$

e.g.

$|Q| = 20$ in NFA, the corresp DFA has state set $\text{Power}(Q)$

size $2^{20} > 1,000,000$

NFA q_0 q_1 q_2 $\vdots$ $\cdots$ q_{19}

Say we in DFA state

$$\{q_0, q_3, q_5, q_{12}, q_{13}, q_{16}\} = \tilde{q}$$

here and we read "b"

$$\delta(\quad, b) \stackrel{?}{=}$$

where can b and ε's take us? from $\tilde{q}$

$\tilde{Q} \leftrightarrow$ is q_0 in? is q_1 in? , ... ,

0, 1 , 0, 1